Optimal Taxation with Political Externalities

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Abstract

When should tax policy be used to influence political donation behavior? In a model of electoral politics where campaign spending is financed by citizen donations, inequality of political influence favoring the "donor class" can arise. Adopting the normative stance that such inequality is undesirable, characterizations of optimal linear and nonlinear taxation of political donations are presented. Sufficient statistics for optimal policy include not only donation demand elasticities, but also the marginal efficacy of campaign spending, and the effect of taxes on the sensitivity of donations to candidate policy platforms. Using numerical simulations, I provide proof-of-concept results showing that this framework can rationalize real world policies such as the nonlinear subsidy schedules present in Canada. These feature generous marginal rates of subsidy on the first dollar of political donations, but the rate of subsidy declines in donation amount.

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Politics in modern representative democracies is a resource-intensive enterprise. Political candidates and parties in these societies must somehow acquire the substantial funds needed to credibly compete for citizens' votes. For many, this state of affairs raises concerns about whether and how the financing of political activity distorts political outcomes relative to some democratic ideal. Perhaps fundraising incentives cause politicians to place disproportionate weight on the policy preferences of wealthy donors, special interest groups, or other actors who provide political funds. Writing about the modern politics in the United States, Overton [2004] suggests that "a small, wealthy, and homogenous donor class... effectively determines which candidates possess the resources to run viable campaigns" and that this has the effect of enhancing the relative political influence of this "donor class" in a way which undermines democratic principles. Scarrow [2018] suggests that such phenomena contravene ideals of political equality; specifically, the principle of "citizen parity", which is the idea that citizens in a democracy ought to have equal "opportunities to influence the political process".

If these concerns are warranted—and deemed normatively important—then political finance activities might be said to produce a kind of externality which calls for a corrective policy response. Scarrow [2018], in a review of the effects of political finance regulation on political equality, highlights a range of possible options including public subsidies for political parties/campaigns, regulation of campaign spending, limiting the size of contributions, or subsidizing small donors. This last type of policy is the focus of this paper, as it motivates a natural set of questions for tax economists. Can "political externalities" be corrected using standard tax policy tools? Should a social planner simply levy Pigouvian taxes/subsidies on certain political finance transactions? What is the marginal social cost/benefit of these transactions?

Existing work in tax analysis does not provide answers to these questions. This is a striking gap in view of the fact that many political finance regulatory measures that have been proposed or implemented around the world make explicit use of the standard tools of tax policy. For example, in Canada, there are substantial tax credits available for political contributions at both the federal and provincial/territorial level [Elections Canada, 2017]. These credits vary in their design but most commonly entail a 75% credit on the first dollar of donations, with a declining marginal rate of subsidy at higher levels of donations. Total individual donations are also capped annually. Existing work in the tax policy literature provide no clear guide to analyzing such policies.

The absence of a formal theoretical treatment of the tax policy implications of "political externalities" seems even more glaring in light of the fact that some prominent public economists have cited concerns about political inequality as justification for instituting a wealth tax and/or raising top marginal income tax rates. Saez and Zucman [2019] argue that wealth inequality generates social costs by inducing inequality of political power (among other mechanisms). They suggest that these costs are externalities analogous to the costs of pollution, and thus corrective taxation of wealth and/or high incomes may be justified. Kopczuk [2019] notes that standard theoretical arguments call for directly addressing the source of an externality through taxation or regulation and suggests that it might be preferable to address concerns that wealth concentration leads to concentration of political power via "suitable reforms of political system"? Scheuer and Slemrod [2020] raise similar concerns. Viewed through the lens of optimal tax theory, these authors are engaged in speculation about whether Sandmo's "Principle or Targeting" or "Additivity Property" applies to some ambiguously defined political externalities. The literature on optimal tax theory has explored the conditions under which this property holds but absent a clear definition of what political externalities are this work provides no guidance this question [Micheletto, 2008].

In this paper, I take a first step towards expanding normative tax analysis to account for concerns about the effects of tax policy on the functioning of democratic institutions. At its core, this theory rests on three components:

1. A tractable model of political institutions, which maps from citizen preferences and actions to equilibrium policy outcomes.
2. A normative criteria for evaluating the equilibrium policy outcomes of the model.
3. A normative thought experiment which specifies which policy instruments are endogenously controlled by the model of political institutions and which are exogenously controlled by a social planner.

Section 1 accomplishes the first task by introducing a simple extension of probabilistic voting models which incorporates campaign spending financed by individual donors. Equilibrium policy choices in this model can be characterized the maximizer of an implicit social welfare function, clarifying how campaign finance motives may influence policy outcomes. In classic probabilistic voting models, a citizen's implicit welfare weight is proportional to their relative importance as a marginal source of votes. In my model, this result is augmented so that a citizen's implicit welfare weight also depends on their relative importance as a *marginal source of donations*, reflecting the candidates' incentive to alter their policy platforms as a way to finance campaign expenditures which can increase their vote share. The candidates weight this new marginal donations incentive in proportion to the *marginal effect of campaign spending* on vote share. This intuitive result may seem straightforward, but it is significant in its own right. The idea that a donor's influence on may not depend on the relative size of their donation so much as their importance as a marginal donor has received little discussion in the political finance literature. As I demonstrate later, this distinction has important implications for what kind of externalities political donations generate and, consequently, for optimal taxation of donations.

Section 2 presents a definition of *political externality* as the as the welfare loss associated with implementing policies chosen by the elections model rather than imposing the political welfare-maximizing policy. This is one normative criteria which can be used to assess equilibrium policy outcomes and compare them against alternative outcomes which would prevail if a particular tax (or other policy) reform were adopted. In this section, I three key mechanisms through which such reforms can alter the political externality: by changing how import different citizens are as a marginal source of donations; by changing voter turnout rates; and, by changing the marginal effect of campaign spending. I give particular focus to the case of a donation ban as a way of sharply illustrating the implications of this framework. I also provide a general characterization of the political externality impact of a policy reforms that influences donation behavior, which serves as an input to the optimal tax analysis of the next section.

Section 3 describes a specific normative thought experiment in which the policy space in the elections model is the extent of redistribution to be implemented in the period after the election. The social planner chooses the tax treatment of political contributions in the period before the election (before the redistribution policy is implemented). Given a fixed budget, they seek to implement a tax/subsidy schedule which minimizes the political externality while also accounting for the direct welfare impact of donation taxation.

Given this setup, I derive a necessary condition characterizing the optimal nonlinear donation tax schedule. Such a tax schedule can influence political externalities through two mechanisms. First, it can alter the amount of campaign spending candidates do. Whether the marginal dollar of campaign spending has a positive or negative effect on political externalities depends on how the spending affects the distribution of implicit welfare weights. For example, if such spending enhances voter turnout amongst relatively high-turnout groups, it may be that marginal spending has a negative effect on political externalities. On the other hand, if inequality of importance as a marginal source of donations is the primary source of political externalities, and there are diminishing marginal returns to campaign expenditure increased campaign spending may have a positive effect, since driving down the marginal effect of campaign spending reduces the candidates' valuation of marginal donations. However these various factors net out, the effect of the campaign spending mechanism on political externalities can be addressed via the standard Pigouvian remedy: a tax/subsidy on political contributions equal to the social cost/benefit of the marginal dollar of campaign spending. This result reflects the fact that this campaign spending effect is a standard atmospheric externality: it depends only on the total donations made and is invariant to the composition of the contributors to that total.

The second way that donation taxation can affect political externalities is by changing the how responsive citizen donations are to the policies candidates propose. This is analogous to the effect that taxation of a specific commodity might have on the relationship between a consumer's willingness-to-pay for a particular product and its quality. By changing this responsiveness of citizen donation behavior—what I call the *donation sensitivity*—taxation can change the relative importance of citizens as a source of marginal donations. It is not obvious whether this mechanism will in general have a positive or negative effect on political externalities: this depends on how the tax affects different types of citizens' donation sensitivity and how relative political influences covaries with such differences. But it is important to note that this effect cannot be treated as a standard atmospheric externality, as it does not depend on the total amount of donations. Indeed, it does not it fit the standard description of a non-atmospheric externality either as it does not directly depend on the donation quantities at all. This result casts doubt on the validity of deriving intuitions about corrective policy for political externalities from the results that pertain to standard types of externalities.

The effect of taxation on political externalities via its effect on donation sensitivity provides a rationale for nonlinear taxation of political contributions. If citizens with differing relative political influence tend to systematically locate on different parts of the donation distribution, a nonlinear tax schedule may allow the planner the flexibility to enhance the donation sensitivity of some groups while reducing it for others, while simultaneously manipulating the level of total donations in order to control political externalities via campaign spending effects. Numerical simulations presented in Section 3.4 confirm this intuition, demonstrating that—at least in a particular example model—the optimal tax on donations is nonlinear, featuring a very generous marginal rate of subsidy at low values of donations, with a declining rate of subsidy as donation amounts increase, culminating in an infinite positive top tax rate on donations. These findings suggest that the framework introduced in this paper can provide a plausible justification for the nonlinear donation subsidy schedules seen in countries like Canada.

In section 4, I consider an important extension of my results. The welfarist definition of political externality I have adopted in this paper does not appropriately capture some normative frameworks which are common in the political finance literature. I consider an alternative definition intended to capture the idea that inequality of political influence has a direct normative cost, irrespective of how it affects the policy outcome that results from the political process. This alternative approach defines the "political externality" as the measured inequality in political influence weights in equilibrium. I show that it is straightforward to extend the optimal tax results of section 3 to this alternative definition.

I conclude by discussing possible extensions of the model and briefly considering the prospects for applied welfare analysis. Empirical application of my optimal tax formulae requires the estimation of novel sufficient statistics which have received no prior discussion or empirical investigation in the political finance literature: the effect donation taxation on donations sensitivity. In fact, it is not enough even to estimate these effects of taxation: applied welfare analysis using my formulae also requires knowing how these effects are differ across citizens with different relative political influence or policy preferences. These high informational requirements suggest that more work is needed to translate the insights of this paper into applied policy recommendations.

The reader should note that this work is meant to be illustrative rather than comprehensive. In detailing one approach to constructing a theory of optimal taxation with political externalities, this paper highlights a set of general issues which must be confronted in order to address this topic. This is a task which demands further attention if tax economists and political finance scholars seek to provide informed insights about how tax policy should be designed when it affects the functioning of political institutions.

1 A Spatial Model of Elections with Individual Donors

Here I develop a probabilistic voting model which accounts for the role of individual donors and campaign expenditures in electoral competition.[1] The probabilistic voting theory literature develops spatial models of elections in which candidates who are uncertain about citizen voting behavior strategically choose policy commitments with the goal of winning votes.[2] Prominent contributions include Hinich et al. [1972, 1973], Hinich [1976, 1978], Lindbeck and Weibull [1987, 1993], and Coughlin [1992]. Banks and Duggan [2005] provide a unified treatment of probabilistic voting models in two candidate majority-rule elections.

Note, the version of the elections model discussed here is highly stylized. This is intended to simplify the discussion of my primary topic of interest: the optimal tax implications of political externalities.[3]

1.1 Setup

Agents Two *candidates* (A and B) compete in an election by simultaneously choosing policy positions. Both candidates announce a *policy position* $x \in X$ where X is a compact and convex (possibly multidimensional) policy space. For any candidate $c \in \{A, B\}$, let x^c be their policy position. Let $-c \equiv \{A, B\} \setminus \{c\}$ denote candidate c's opponent. Both candidates also engage in campaign activities designed to raise voter turnout amongst their supporters. These activities are financed by donations the candidates receive.[4]

The voters and donors in the election are a unit mass of *citizens*. Each citizen is characterized by a *type* $\theta \in \Theta$ and a *preference shock* $\xi \in \mathbb{R}$ which jointly determine their political behavior. Citizen types are distributed

[1] To my knowledge, the extension of probabilistic voting theory I present here has no precedent in the literature with the notable exception of Hettich et al. [1999, p. 136], who provide a very brief sketch of such a model in a their comprehensive treatise on the political economy of taxation, which makes extensive use of probabilistic voting models.

[2] Candidate uncertainty about citizen behavior is what distinguishes these models from the other predominant framework for modeling strategic candidate policy choice: deterministic or "Downsian" models of elections. These assume candidates know citizen policy preferences with certainty [Hotelling, 1929, Downs, 1957, Black, 1958].

[3] In ongoing work, I am exploring a more general version of the model.

[4] This is just one way to introduce campaign finance motives into a probabilistic voting model. In ongoing work I am exploring the implications of alternative assumptions regarding the role of money in politics.

according to some measure μ. Citizen preference shocks are independent of type and distributed according to G, a symmetric, mean zero distribution.

I will sometimes refer to a type θ citizen with a preference shock ξ as a (θ, ξ)-citizen.

Policy Preferences All type θ citizens are assumed to have the same preferences over the policy that will be implemented after the election. These preferences are represented by a *policy utility function* $u_\theta : X \to \mathbb{R}$ which is bounded above on X.

I assume that citizen political behavior on depends on policy utilities through the difference between candidate policy utilities:[5]

$$\Delta u_\theta \left(x^c, x^{-c}\right) \equiv u_\theta \left(x^c\right) - u_\theta \left(x^{-c}\right).$$

For convenience, I will sometimes use the shorthand $\Delta u_\theta^c \equiv \Delta u_\theta \left(x^c, x^{-c}\right)$. Note, these definitions imply that $\Delta u_\theta^c = -\Delta u_\theta^{-c}$.

Candidate Quality Citizens make political decisions based on their subjective perception of the relative quality of the two candidates. In part, quality is derived from candidate policy platforms: the larger is Δu_θ^c, the higher quality a type θ citizen assigns to candidate c. However, perceived quality is not based on policy preferences alone. Citizens' preference shock ξ also plays a role in their decision. In particular, for a (θ, ξ)-citizen the *perceived quality* of candidate c is

$$Q_\theta^c\left(\xi\right) \equiv \Delta u_\theta^c + \xi^c$$

where $\xi^A \equiv \xi$ and $\xi^B \equiv -\xi$. Note, since G is mean zero and symmetric, both ξ^c is distributed according to G for both $c \in \{A, B\}$. Note that given this definition we have $Q_\theta^A\left(\xi\right) = -Q_\theta^B\left(\xi\right)$.

Preferred Candidate Candidate c is defined as the *preferred candidate* of (θ, ξ)-citizen if and only if the perceived quality of c is positive for such citizens:

$$Q_\theta^c\left(\xi\right) > 0.$$

[5] In ongoing work, I am exploring the implications of relaxing this assumption.

Notice, given the definition of $Q^c_\theta(\xi)$, a citizen can only prefer one candidate. The probability that some type θ citizen prefers candidate c is:[6]

$$\begin{aligned}\Pr\{\Delta u^c_\theta + \xi^c > 0\} &= 1 - G(-\Delta u^c_\theta) \\ &= G(\Delta u^c_\theta).\end{aligned}$$

I will often refer to the set of citizens that prefer candidate c as candidate c's *supporters.*

Voting Decision If a citizen votes, they vote for their preferred candidate. However, voting is costly, so a citizen will not necessarily vote for their preferred candidate. In particular, suppose that a type θ supporter of candidate c will cast a vote for candidate c with probability $\eta_\theta(D^c)$, where D^c is the total donations candidate c receives in the election. For all θ, let $\eta_\theta : \mathbb{R}_+ \to [0,1]$ be a strictly increasing and strictly concave function.[7] Notice, I allow this turnout probability function η_θ to vary by type. This reflects the possibility that individual policy preferences may be correlated with other factors that influence the baseline probability of voting and the effect of campaign spending on the probability of voting.

Donation Decisions In addition to making voting decisions, citizens decide whether to make monetary contributions to candidates' campaigns, and if so, how much to contribute. Citizens receive some warm-glow utility from donating to their preferred candidate. I postpone a full discussion of the microfoundations of citizen donation behavior to section 3. For the moment, I simply assume that some

Suppose that a (θ, ξ)-citizen donates $\delta_\theta(Q^c_\theta(\xi))$ to candidate c, where $\delta_\theta : \mathbb{R} \to \mathbb{R}_+$ is a type-specific function describing how perceived quality changes donation demand. Assume that $\delta_\theta(\cdot)$ is weakly increasing in perceived quality and citizens donate only make positive donations to their preferred candidate: $\delta_\theta(Q) = 0$ for all $Q \leq 0$. Like the turnout probability functions, donation functions vary by type, reflecting the possibility that policy preferences may be correlated with other factors that influence an individual's willingness to donate money to political campaigns.

Given these assumptions, the expected donation a type θ citizen makes to candidate c when $\Delta u^c_\theta = \Delta u$ is

$$\bar{\delta}_\theta(\Delta u) \equiv \int_{-\Delta u}^{\infty} \delta_\theta(\Delta u + \xi)\, \mathrm{d}G(\xi).$$

[6]The second equality follows from symmetry of G.

[7]I am adopting this approach to modeling costly voting for simplicity. In ongoing work I am exploring alternative assumptions about the use of campaign funds and the voter turnout decision. This includes, for example, the case where voter turnout decisions depend on Δu^c_θ and cases where donation can directly influence the choice of preferred candidate and citizen donation behavior.

Aggregate Behavior If the candidates adopt policy positions $\left(x^{A}, x^{B}\right)$, then the total donations to candidate c are

$$D^{c}\left(x^{A}, x^{B}\right) \equiv \int \bar{\delta}_{\theta}\left(\Delta u\right) \mu\left(\mathrm{d}\theta\right). \tag{1}$$

Let $\eta_{\theta}^{c}\left(x^{A}, x^{B}\right) \equiv \eta_{\theta}\left(D^{c}\left(x^{A}, x^{B}\right)\right)$ be the turnout probability for candidate c's type θ supporters given the policy positions $\left(x^{A}, x^{B}\right)$. The probability that a given citizen of type θ votes for candidate c is[8]

$$P_{\theta}^{c}\left(x^{A}, x^{B}\right) \equiv \eta_{\theta}^{c}\left(x^{A}, x^{B}\right) \cdot G\left(\Delta u_{\theta}^{c}\left(x^{A}, x^{B}\right)\right) \tag{2}$$

and thus the share of citizens voting for candidate c is

$$P^{c}\left(x^{A}, x^{B}\right) \equiv \int P_{\theta}^{c}\left(x^{A}, x^{B}\right) \mu\left(\mathrm{d}\theta\right). \tag{3}$$

Note that P^{c} and D^{c} are deterministic because there is a continuum of citizens each independently drawing a preference shock from G.

Candidates' Problem Candidate c chooses their policy position to maximize their plurality of votes in the election:

$$\max_{x \in X} \left\{P^{c}\left(x^{c}, x^{-c}\right) - P^{-c}\left(x^{-c}, x^{c}\right)\right\}. \tag{4}$$

Let $P\ell^{c}\left(x^{c}, x^{-c}\right) \equiv P^{c}\left(x^{c}, x^{-c}\right) - P^{-c}\left(x^{-c}, x^{c}\right)$ denote candidate c's objective function.

Note, candidates do not seek to maximize their probability of winning the election but rather the difference between the votes they receive and the votes their opponent receives. This type of objective function is common in the probabilistic voting literature [Banks and Duggan, 2005]. However, because I have assumed a continuum of citizens with independently distributed partisan biases, any equilibrium under this objective function will also be an equilibrium under a probability of winning objective, but not vice versa.[9]

For notational convenience, in the remainder of the paper I will often suppress the dependence of the functions defined above on candidate policy choices (e.g. writing η_{θ}^{c} instead of $\eta_{\theta}^{c}\left(x^{A}, x^{B}\right)$).

[8] The probability a type θ citizen prefers candidate c is

$$\begin{aligned}\operatorname{Prob}\left\{\Delta u_{\theta}^{c} + \xi^{c} > 0\right\} &= 1 - G\left(-\Delta u_{\theta}^{c}\right)\\ &= G\left(\Delta u_{\theta}^{c}\right)\end{aligned}$$

where the second equality follows from symmetry of G. Multiplying this by candidate c's turnout probability gives P_{θ}^{c}.

[9] This is because the vote shares in this model are deterministic. Suppose that in the case of a tie each candidate has a $\frac{1}{2}$ probability of winning and otherwise the candidate with the highest vote share wins. If there exists some policy position pair where one candidate has a higher vote share and the losing candidate can deviate to an alternate policy position which would increase their vote share but not by enough to change the election outcome. This would be a profitable deviation under the vote share objective but not under the probability of winning objective.

1.2 Equilibrium Policy and Implicit Welfare Functions

The election game is a static game where the two candidates simultaneously choose policy positions. A pair of policy positions $\left(x^{\star A}, x^{\star B}\right)$ is an *electoral equilibrium* of this game if

$$x^{\star A} = \arg\max_{x \in X} \left\{ P^A\left(x, x^{\star B}\right) - P^B\left(x, x^{\star B}\right)\right\} \tag{5}$$

and

$$x^{\star B} = \arg\max_{x \in X} \left\{ P^B\left(x^{\star A}, x\right) - P^A\left(x^{\star A}, x\right)\right\} \tag{6}$$

for all $c \in \{A, B\}$.[10] A *convergent equilibrium* is an electoral equilibrium in which $x^{\star A} = x^{\star B} = x^{\star}$. This is the standard equilibrium definition used in the probabilistic voting literature [Banks and Duggan, 2005].

Candidate policy choices in an electoral equilibrium can be characterized using a simple first-order condition approach.[11] Let $\mathbf{x}^{\star} \equiv \left(x^{\star A}, x^{\star B}\right)$ be an electoral equilibrium and $g(\xi) \equiv G'(\xi)$ be the preference shock probability distribution function. The first-order condition characterizing candidate c's choice is

$$\underbrace{\int \left[g\left(\Delta u_\theta^c\right)\eta_\theta^c + \psi^c \bar{\delta}_\theta'\left(\Delta u_\theta^c\right)\right] \nabla_{x^c} u_\theta^c\, \mu\left(\mathrm{d}\theta\right)}_{=\nabla_{x^c} P^c} + \underbrace{\int \left[g\left(\Delta u_\theta^{-c}\right)\eta_\theta^{-c} + \psi^{-c} \bar{\delta}_\theta'\left(\Delta u_\theta^{-c}\right)\right] \nabla_{x^c} u_\theta^c\, \mu\left(\mathrm{d}\theta\right)}_{=\nabla_{x^c} P^{-c}} = 0 \tag{7}$$

where

$$\psi^c \equiv \int \eta_\theta'\left(D^c\right) G\left(\Delta u_\theta^c\right) \mu\left(\mathrm{d}\theta\right)$$

is the *marginal effect of campaign spending* for candidate c. This parameter measures the marginal impact of an additional dollar of donations (and resulting expenditures) on candidate c's objective function.[12]

Equation (7) provides a simple, intuitive description of how campaign finance issues can influence policy outcomes. Political candidates in this model choose a location in the policy space taking into account not only how changes in their policy influence their plurality, but also how such changes influence the donations that they will receive. They value these marginal donations in proportion to the marginal effect of campaign spending.

[10] Note, this is simply a generalized Nash equilibrium. See Facchinei and Kanzow [2007] for a review of generalized Nash games.

[11] Here and for the remainder of the paper, I ignore the assumptions required to enable this, as exploring the technical properties of this model is not my primary objective here.

[12] This description of the marginal effect of campaign spending is very specific to this variant of my elections model. In ongoing work, I am exploring the nature of this parameter in a more general version of the model and in alternative special cases of the general model. The precise form of this parameter varies substantially depending on the assumptions made about campaigning technology.For example, when campaign spending can influence donation behavior the marginal effect of campaign spending includes feedback loops between donation behavior and spending (i.e. a dollar of spending of additional spending may induce additional donations to the candidate, which induce more spending, which induce more donations, etc) and effects of campaign spending on the behavior on donations to the candidate's opponent (i.e. a dollar of spending of additional spending may reduce donations to the opponent, which reduces their spending, which further reduces donations, etc).

To be more explicit about how these issues translate into equilibrium policy positions, note that equation (7) implies for any electoral equilibrium $\mathbf{x}^{\star}$ candidate c's policy choice will also satisfy

$$x^{\star c} = \arg\max_{x \in X} \left\{ \int \gamma_{\theta}^{c}\left(\mathbf{x}^{\star}\right) u_{\theta}\left(x\right) \mu\left(\mathrm{d}\theta\right) \right\} \tag{8}$$

where

$$\gamma_{\theta}^{c}\left(\mathbf{x}^{\star}\right) \equiv \underbrace{\sum_{k \in \{c,-c\}} \eta_{\theta}^{k}\left(\mathbf{x}^{\star}\right) g\left(\Delta u_{\theta}\left(x^{\star k}, x^{\star -k}\right)\right)}_{\text{value as marginal source of votes}} + \underbrace{\sum_{k \in \{c,-c\}} \psi^{k}\left(\mathbf{x}^{\star}\right) \bar{\delta}_{\theta}'\left(\Delta u_{\theta}\left(x^{\star k}, x^{\star -k}\right)\right)}_{\text{value as marginal source of donations}} \tag{9}$$

is the *political influence* of type θ citizens for candidate c in the equilibrium $\mathbf{x}^{\star}$. Thus, candidate c's policy choice in an electoral equilibrium can be described as the maximizer of a weighted average of individual policy utilities, where the weight assigned to the policy preferences of type θ citizens is their political influence $\gamma_{\theta}^{c}\left(\mathbf{x}^{\star}\right)$. Notice, as equation (9) makes clear, the political influence of citizens is specific to a particular equilibrium of the election game. Thus, the maximization problem in equation (8) is not equivalent to the candidate's problem presented in equation (5), but rather provides an implicit characterization of a given equilibrium. Given this, I will sometimes refer to the objective function in equation (8) as the *implicit welfare function* at $\mathbf{x}^{\star}$. This characterization is useful for building intuition about how campaign finance issues influence equilibrium policy outcomes.

Equations (8) and (9) imply that in any electoral equilibrium, candidate c weights the policy preferences of type θ citizens in proportion to their importance as a marginal source of votes and as a *marginal source of donations*. As well, it is important to note that the candidate not only cares about how their policy position affects the votes and donations they receive, but also how it affects the votes and donations their opponent receives. The multipliers $\psi^{c}\left(\mathbf{x}^{\star}\right)$ and $\psi^{-c}\left(\mathbf{x}^{\star}\right)$ reflect the marginal value of donations in terms of "buying" votes, and thus determine the relative importance of the marginal vote and marginal donation incentives for the candidate's policy position.

This simple result already highlights an important issue which has received little attention in the public and academic discourse on political finance issues. Notice, the model does not imply that citizens who donate more money will have disproportionate political influence: rather it suggests that citizens who will increase/decrease their donations by relatively large amounts in response to marginal policy changes are relatively influential. Thus, it is possible that large donor who strongly favors one candidate's policy position could actually have relatively little influence on the candidate's policy choice at the margin. In the language of the model, equilibrium policies favor citizen types with large values of $\bar{\delta}_{\theta}'\left(\Delta u_{\theta}^{A}\right)$ and $\bar{\delta}_{\theta}'\left(\Delta u_{\theta}^{B}\right)$, not necessarily those with large values of $\bar{\delta}_{\theta}\left(\Delta u_{\theta}^{A}\right)$ and $\bar{\delta}_{\theta}\left(\Delta u_{\theta}^{B}\right)$.

This fact will prove important to my later results so it will be helpful to define some terminology. For a (θ, ξ)-citizen who prefers candidate c I call $\delta'_\theta(\Delta u^c_\theta + \xi^c)$ their *donation sensitivity.* Type θ citizens as a group also have an *aggregate donation sensitivity* for candidate c: $\bar{\delta}'_\theta(\Delta u^c_\theta)$.

1.3 Convergent Equilibria

Note that equation (9) implies that the candidates will have identical sets of political infuence weights. This results from the concern each candidate has for the impact of their policy choice on their opponent's vote share and total donations. Symmetry of welfare weights implies that if there exists some electoral equilibrium, there exists a convergent equilibrium.[13] implies that there exists a convergent equilibrium policy profile $(x^{\star A}, x^{\star B})$ where $x^{\star A} = x^{\star B} = x^\star$. In this case, the candidate first-order condition becomes

$$\int \gamma_\theta \nabla_x u_\theta(x)\, \mu(\mathrm{d}\theta) = 0$$

where political influence is

$$\gamma_\theta \equiv 2\eta_\theta(D)\, g(0) + \left[\int \eta'_\theta(D)\, \mu(\mathrm{d}\theta)\right] \bar{\delta}'_\theta(0), \tag{10}$$

and total donations per candidate are

$$D = \int \bar{\delta}_\theta(0)\, \mu(\mathrm{d}\theta).$$

Equivalently, the equilibrium policy can be characterized by as the maximizer of a simplified implicit weighted utilitarian maximization problem:

$$x^\star = \arg\max_{x \in X} \left\{ \int \gamma_\theta u_\theta(x)\, \mu(\mathrm{d}\theta) \right\}. \tag{11}$$

For simplicity, throughout the rest of the paper I assume that the political process results in a convergent equilibrium, leaving a discussion of divergent equilibria to future work.

2 Defining Political Externalities

Consider a society where an election of the kind described in section 1 is held in order to select a policy. Suppose that the resulting policy, $x^\star \in X$, is a convergent equilibrium as described by equation (11).

[13] To see this, suppose that (x, x') is an electoral equilibrium. This implies that x is the maximizer of candidate A's implicit social welfare function. By symmetry of welfare weights and because welfare weights are independent of candidate policy choices, x is also a maximizer of candidate B's implicit social welfare function. Hence, (x, x) is an electoral equilibrium.

In this section, I propose a welfarist measure of the *political externality* associated with $x^{\star}$: the gain in social welfare that would result from implementing the welfare-maximizing policy instead of the equilibrium policy. To help to illustrate the implications of this framework, I discuss how a ban on donations would change the political externality. Finally, I provide a general characterization of the political externality effects campaign finance reforms that alter donation behavior.

2.1 A Welfarist Approach to Political Externalities

Consider the utilitarian social welfare function

$$W(x) \equiv \int u_{\theta}(x)\,\mu(\mathrm{d}\theta), \tag{12}$$

which weights the policy preferences of all citizens equally. I assume that this function provides the normatively relevant measure of the social welfare associated with the policy outcome of the political process. Arguably, this choice of benchmark is consistent with the notion that an ideal democracy would feature perfect equality of political influence.

Letting $x^U \in X$ be the policy that maximizes $W(x)$ and let $x^{\star} \in X$ be a convergent equilibrium of the election model described in section 1. A natural definition of the *political externality* associated with $x^{\star}$ is

$$PE(x^{\star}) \equiv W\left(x^U\right) - W(x^{\star}), \tag{13}$$

the loss in welfare that results from implementing the political equilibrium policy $x^{\star}$ rather than the utilitarian-optimal policy x^U.

Equilibrium policy will generally differ from the welfare-maximizing policy ($x^U \neq x^{\star}$) because—as discussed in section 1.2—the political process weights the policy preferences of some citizens more highly than others. Thus, absent perfect equality of political influence, the political externality will generally be positive: $PE(x^{\star}) > 0$.

2.2 Donation Bans and a Political Theory of the Second-Best

Consider a radical campaign finance reform proposal: banning all campaign donations. Would such a reform reduce the size of the political externality?

If eliminating campaign donations resulted in perfect equality of political influence, then such a reform would result in the political process implementing a the utilitarian-optimal policy x^U. However, this is not necessarily the outcome that the model should lead us to expect. Evaluating equation (10) with $D = 0$ and $\bar{\delta}'_\theta(0)$ for all θ gives us the political influence weights under a donation ban:

$$\tilde{\gamma}_\theta \equiv 2g(0)\,\eta_\theta(0)\,.$$

These weights are constant if and only if voter turnout rates are constant across citizen types in the absence of campaign spending. Thus, we should generally expect the equilibrium policy under a donation ban

$$\tilde{x} = \arg\max_{x\in X}\left\{\int \eta_\theta(0)\,u_\theta(x)\,\mu(\mathrm{d}\theta)\right\}$$

will also differ from the the utilitarian-optimal policy x^U. Thus, as long as campaign spending is not the sole cause of turnout differences across citizens, the political externality will remain positive with a donation ban in place: $PE(\tilde{x}) > 0$.

Suppose that it is indeed the case that, absent donations, inequality of political influence already exists due to turnout differences. Will the political externality with a donation ban in place ($PE(\tilde{x})$) be higher or lower than the political externality without one ($PE(x^\star)$)? The answer is not obvious.

Consider how lifting a donation ban changes political influence weights. Allowing donations to occur introduces a new source of inequality of political influence, through inequality of donation sensitivity. Let us suppose that donation sensitivity is relatively high for precisely those citizens who had relatively high turnout under the donation ban ($\eta_\theta(0)$). All else equal, this increases influence of citizens who were already relatively influential, and thus would be expected to increase the political externality. However, this is not the only thing that changes with the lifting of the donation ban; campaign spending also increases. This spending (weakly) increases voter turnout rates of all citizen types. Suppose that campaign spending has a larger effect on turnout among citizens with relatively low turnout. All else equal, this increases the political influence of relatively low influence citizens, and would generally be expected to decrease the political externality.[14]

Whether lifting a donation ban increases or decreases the political externality depends on the net impact of these two counterveiling effects. This scenario highlights provides an example of a more general point: when there are multiple sources of inequality of political influence, reforms which reduce inequality of influence through only one mechanism may not necessarily be welfare-improving and, indeed, can actually decrease welfare. This point is a kind of political analogue of one made by Lipsey and Lancaster [1956] regarding economic policy. They note that while a policy reform which distorts economic behavior is efficiency-decreasing

[14]Note, in this discussion, I have assumed that higher inequality of political influence implies a higher political externality. However, this only holds true if policy preferences systematically vary with relative political influence of citizens in such a way that policies which are better for the relatively influential citizens are necessarily worse for the relatively low influence citizens.

if the economy is efficiency, the same reform can be efficiency-enhancing if the economy is inefficient due to the presence of some pre-existing distortion.

The political externality impact of a donation ban would likely be even less clear in a richer model of political participation, where additional sources of inequality of political influence might be present. For example, modern political campaigns are highly dependent on volunteer labor. If such attracting this labor is valuable to candidates, it may create distortions of political influence through a mechanism very similar to the donation sensitivity mechanism in the model of section 1. A donation ban would not eliminate this source of influence inequality and whether on whether allowing campaign spending would increase or decrease this type of inequality is not a priori obvious.[15,16]

It is important to note that the discussion above provides an incomplete analysis of the normative implications of a donation ban. Even if a ban would reduce the political externality, this does not necessarily mean that a ban would be welfare-improving. This is because the political externality effect of the ban do not include the welfare losses experienced by citizens in period 1 who who want to donate to their preferred political candidate but will be prevented from doing so. In the next section's optimal donation tax analysis, I account for these additional welfare costs.

2.3 Political Externality Effects of Reforms

Given that it is unclear whether or not a donation ban is optimal campaign finance regulation, the next section considers more subtle forms of policy; specifically, donation taxes and subsidies. The political externality effects of these policies will provide an important input to my analysis of optimal donation taxation. In anticipation of that discussion, here I will discuss the political externality effect of reforming some abstract campaign finance regulation indexed by $\kappa \in \mathbb{R}$. Suppose that κ only influences the political system through its impact on citizen donation behavior. Donation taxes and subsidies are one example of such a policy, but other policies—such as donation caps—could also be analyzed using this framework.

The political externality effect of a marginal increase of κ is

$$\frac{\mathrm{d}PE}{\mathrm{d}\kappa} = -\int \nabla_x u_\theta\left(x^\star\right)\mu\left(\mathrm{d}\theta\right) \cdot \frac{\mathrm{d}x^\star}{\mathrm{d}\kappa}$$

[15] This would depend on factors such as whether spending and volunteer labor are complements or substitutes at the margin.

[16] This "political theory of the second best" may have significance beyond the context of campaign finance reform. Proposals for democratic reform often suggest changing a single aspect of the political system in a "more democratic" direction. In principle, this narrow focus could result in reform proposals that inadvertently enhance political inequality. I leave an investigation of the broader application of this principle to future work.

where, by the implicit function theorem,

$$\frac{\mathrm{d}x^\star}{\mathrm{d}\kappa} = -\left[\int \gamma_\theta \nabla_{xx} u_\theta\left(x^\star\right)\mu\left(\mathrm{d}\theta\right)\right]^{-1}\int \nabla_x u_\theta\left(x^\star\right)\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}\kappa}\mu\left(\mathrm{d}\theta\right).$$

Note, this expression depends on the Hessian of policy utility for each citizen type $\nabla_{xx} u_\theta\left(x^\star\right)$.

Notice, the political externality effect of the reform can be described as a particular aggregation of the way the reform changes the political influence of citizens,

$$\frac{\mathrm{d}PE}{\mathrm{d}\kappa} = \int \tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}\kappa}\mu\left(\mathrm{d}\theta\right), \tag{14}$$

where

$$\tau_\theta^{PE} \equiv \left[\int \nabla_x u_{\bar\theta}\left(x^\star\right)\mu\left(\mathrm{d}\bar\theta\right)\right]\cdot\left[\int \gamma_{\bar\theta}\nabla_{xx} u_{\bar\theta}\left(x^\star\right)\mu\left(\mathrm{d}\bar\theta\right)\right]^{-1}\nabla_x u_\theta\left(x^\star\right)$$

is the *political externality wedge* of type θ citizens: the marginal effect that increasing the political influence of a type θ citizen has on the political externality (holding constant the weights of all other citizens).

Political Influence Effects of Reform

Equation (14) formalizes the intuitive idea that the political externality effects of a reform depend on how it alters the political influence of citizens. There are three mechanisms through which this occurs:

$$\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}\kappa} = \underbrace{\psi\frac{\mathrm{d}\bar\delta_\theta'\left(0\right)}{\mathrm{d}\kappa}}_{\text{sensitivity effect}} + \underbrace{\frac{\mathrm{d}\psi}{\mathrm{d}\kappa}\bar\delta_\theta'\left(0\right)}_{\text{marginal value of funds effect}} + \underbrace{2g\left(0\right)\eta_\theta'\left(D\right)\frac{\mathrm{d}D}{\mathrm{d}\kappa}}_{\text{voter turnout effect}}. \tag{15}$$

The most intuitively obvious of these is the *sensitivity effect*: all else equal, changing the aggregate donation sensitivity of a group of citizens changes their political influence weight. This effect captures the intuitive idea that a reform which reduces the marginal donations candidates can acquire by making additional policy concessions to group θ will reduce the weight the candidate puts on that group's policy preferences. The effect is proportional to the marginal efficacy of campaign spending, which determines how many additional votes a candidate could gain by spending those marginal donations.

Of course, the marginal efficacy of campaign spending is also impacted by the reform. The *marginal value of funds effect* in equation 15—which captures this mechanism—is proportional to the effect of the reform on the return to marginal campaign spending,

$$\frac{\mathrm{d}\psi}{\mathrm{d}\kappa} = \int \eta''_{\theta}\left(D\right)\mu\left(\mathrm{d}\theta\right)\frac{\mathrm{d}D}{\mathrm{d}\kappa}. \tag{16}$$

This effect results from the fact that a reform which alters donation behavior may change the aggregate amount of donations campaigns receive ($\frac{\mathrm{d}D}{\mathrm{d}\kappa}$) and, consequently, change the amount of campaign spending that occurs. Because I have assumed diminishing returns to campaign spending ($\eta''_{\theta} < 0$) this implies that a reform which reduces total donations will drive up the marginal effect of campaign spending:

$$\frac{\mathrm{d}\psi}{\mathrm{d}\kappa} > 0 \iff \frac{\mathrm{d}D}{\mathrm{d}\kappa} = \int \frac{\mathrm{d}\bar{\delta}_{\theta}\left(0\right)}{\mathrm{d}\kappa}\mu\left(\mathrm{d}\theta\right) < 0. \tag{17}$$

As equation 15 shows, a higher marginal value of spending increases political influence for all citizen types in proportion their aggregate donation sensitivity. The marginal value of funds effect captures an important consequence of certain reforms that aim to reduce the disproportionate influence of the "donor class". This disproportionate influence stems from politicians' incentive to acquire additional campaign funds, as summarized by the marginal value of campaign funds. All else equal, any reform which reduces total donations to the candidates will drive up marginal value of campaign funds. Thus, a reform which reduces inequality in donations sensitivity ($\bar{\delta}'_{\theta}\left(0\right)$) across groups but simultaneously decreases total donations can actually increase total inequality of political influence, because the resulting increase the marginal value of spending (ψ) increases the importance of donation sensitivity inequality to overall inequality of political influence.

The final mechanism through which a reform can impact political influence is through its impact on voter turnout rates. The *voter turnout effect* in equation 15 captures this mechanism. For type θ citizens, the effect is proportional to the change in their voter turnout rate as a result of the reform

$$\frac{\mathrm{d}\eta_{\theta}\left(D\right)}{\mathrm{d}\kappa} = \eta'_{\theta}\left(D\right)\frac{\mathrm{d}D}{\mathrm{d}\kappa}.$$

Notice, this effect captures another potential unintended consequence of regulatory reforms that alter total campaign expenditures: they may alter the pattern of turnout rates across citizens. As noted in section 2.2's discussion of a donation ban, the effect of total campaign spending inequality in turnout rates is generally ambiguous however, if the marginal effect of campaign spending is higher for low turnout citizen types we might expect that campaign spending generally reduces inequality in turnout rates.

An alternate presentation of the way a reform changes the political influence of type θ citizens pools together the marginal value of funds and voter turnout effects under one heading:

$$\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}\kappa} = \underbrace{\psi\frac{\mathrm{d}\bar{\delta}'_\theta(0)}{\mathrm{d}\kappa}}_{\text{sensitivity effect}} + \underbrace{\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\frac{\mathrm{d}D}{\mathrm{d}\kappa}}_{\text{campaign spending effect}}, \tag{18}$$

where $\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D} \equiv \left(2g(0)\,\eta'_\theta(D) + \bar{\delta}'_\theta(0)\int \eta''_\theta(D)\,\mu(\mathrm{d}\theta)\right)$. In the optimal tax analysis of the next section this division of the political externality effects of a reform will prove highly useful, because whereas the effects of taxation which occur through the campaign spending mechanisms can be treated similarly to the traditional case of atmospheric externalities, political externality effects of taxation occurring through the sensitivity mechanism cannot be treated this way.

3 Optimal Taxes and Subsidies on Political Donations

In this section, I present a simple framework for incorporating "political externalities" into the objective function of a social planner who chooses the rate of a linear tax on political contributions. Throughout this section, I will continue to work with the example environment described in section 3.1.

3.1 A Simple Example

To conduct my analysis it will be useful to have a simple example to work with. Consider a two-period model of a democratic society. In each period, a type θ citizen receives an exogenous, type-specific income y_θ.[17] During the first period, no redistribution occurs but an election is held to decide on a redistributive policy to be implemented in the second period. The policy space in this election is a linear tax and transfer program. That is to say, if the winning candidate proposes a tax rate of $t_y \in [0,1]$, then a type θ agent will have a second period consumption of

$$(1-t_y)\,y_\theta + t_y\bar{y}$$

where $\bar{y} \equiv \int y_\theta\mu(\mathrm{d}\theta)$.

[17] Assuming that income is exogenous helps to simplify the exposition of my optimal donation taxation results however, extending the model to account for behavioral responses to taxation is possible.

Policy Preferences and Candidate Quality

Suppose candidate c proposes a tax rate t_y^c as their platform for the election. A type θ citizen's policy utility from t_y^c is the utility they receive in period 2 under this tax rate:

$$u_\theta\left(t_y^c\right) = v\left(y_\theta + t_y^c\left(\bar{y} - y_\theta\right)\right),$$

where $v'(Y) > 0$ and $v''(Y) < 0$ for all $Y \geq 0$. As described in section 1, each each type θ citizen evaluates assigns a quality to the two candidate based on their policy preferences and some preference shock ξ,

$$Q_\theta^c(\xi) = u_\theta\left(t_y^c\right) - u_\theta\left(t_y^{-c}\right) + \xi^c.$$

They vote for the candidate with the higher (positive) quality and must also decide whether to turnout to vote for that candidate as well as what fraction of their period one income they would like to donate to that candidate.

Donations

Regarding donations, I assume that a type θ citizen with a perceived quality for candidate c of Q donates

$$\delta\left(y_\theta, Q\right) \equiv \begin{cases} \arg\max_\delta \phi\left(y_\theta - \delta - \mathcal{T}_\delta\left(\delta\right),\, \delta;\, Q\right) & \text{if } Q > 0 \\ 0 & \text{if } Q \leq 0 \end{cases} \tag{19}$$

to candidate c. The utility function being maximized in the case where $Q > 0$, $\phi(\cdot, \cdot; Q)$, is a standard neoclassic utility function for all Q (i.e. strictly increasing and concave in consumption and donations). Notice, this demand function depends not only on income and perceived quality, but also on a (potentially nonlinear) donation tax schedule $\mathcal{T}_\delta(\cdot)$.

Suppose that a citizen with income $y > 0$ and perceived quality $Q > 0$ chooses a donation amount δ. The marginal net benefit of donating for such a citizen (normalized in terms of consumption) is

$$\mathcal{F}(\delta; y, Q) \equiv MRS\left(y - \delta - \mathcal{T}(\delta),\, \delta;\, Q\right) - \left(1 + \mathcal{T}'(\delta)\right), \tag{20}$$

where the marginal rate of substitution between donations and other consumption is

$$MRS\left(C,\, \delta;\, Q\right) \equiv \frac{\partial\phi(C, \delta; Q)/\partial\delta}{\partial\phi(C, \delta; Q)/\partial C}. \tag{21}$$

Citizens will want to donate until the net marginal benefit of donating is zero, and thus the donation demand function $\delta(Q, y)$ is implicitly characterized by the first-order condition

$$\mathcal{F}(\delta(y, Q); y, Q) = 0. \tag{22}$$

I impose some restrictions on the marginal rate of substitution and the tax schedule in order to ensure that donation demand has certain properties.

Assumption 1 (Second-Order Condition)**.** *For all $y > 0$, and $Q > 0$, the derivative of the marginal net benefit function*

$$\frac{\partial \mathcal{F}(\delta; y, Q)}{\partial \delta} = \frac{\partial MRS(y - \delta - \mathcal{T}(\delta), \delta; Q)}{\partial \delta} - (1 + \mathcal{T}'(\delta)) \frac{\partial MRS(y - \delta - \mathcal{T}(\delta), \delta; Q)}{\partial C} - \mathcal{T}''(\delta), \tag{23}$$

must be negative when evaluated at donation demand $\delta(y, Q)$. That is, donation demand satisfies the second-order $\frac{\partial \mathcal{F}(\delta(y,Q);y,Q)}{\partial \delta} < 0$.

This assumption ensures that donation demand has continuous comparative statics, which can be characterized using the implicit function theorem. Notice, this condition not only restricts preferences to be convex, it also requires that the donation tax schedule is "not too concave" (i.e. that $\mathcal{T}''_{\delta}$ is not too negative).

Assumption 2 (Single Crossing Condition)**.** *For all $C, \delta > 0$, and $Q > 0$, the marginal rate of substitution is strictly increasing in quality: $\frac{\partial MRS(C, \delta; Q)}{\partial Q} > 0$.*

Together with the second-order condition, this assumption ensures that donation sensitivity is positive (i.e. donations are increasing in perceived quality):

$$\frac{\partial \delta(y, Q)}{\partial Q} = \frac{\partial MRS(y-\delta(y,Q)-\mathcal{T}(\delta(y,Q)), \delta(y,Q); Q) / \partial Q}{-\partial \mathcal{F}(\delta(y,Q);y,Q) / \partial \delta} > 0. \tag{24}$$

Assumption 3 (*Donations are a Normal Good*)**.** *For all $C, \delta > 0$, and $Q > 0$, the marginal rate of substitution is increasing in consumption: $\frac{\partial MRS(C, \delta; Q)}{\partial C} > 0$.*

Together with the second-order condition, this assumption ensures that donations are increasing in income:

$$\frac{\partial \delta(y, Q)}{\partial y} = \frac{\partial MRS(y-\delta(y,Q)-\mathcal{T}(\delta(y,Q)), \delta(y,Q); Q) / \partial C}{-\partial \mathcal{F}(\delta(y,Q);y,Q) / \partial \delta} > 0. \tag{25}$$

Assumption 4 (Donation Sensitivity is Increasing in Income)**.** *For all $y > 0$, and $Q > 0$, suppose that*

$$\frac{\partial}{\partial y}\left[\frac{\partial MRS(y-\delta-\mathcal{T}(\delta),\delta;Q)/\partial Q}{-\partial\mathcal{F}(\delta;y,Q)/\partial\delta}\right]_{\delta=\delta(y,Q)}>0 \qquad \textit{and} \qquad \frac{\partial}{\partial \delta}\left[\frac{\partial MRS(y-\delta-\mathcal{T}(\delta),\delta;Q)/\partial Q}{-\partial\mathcal{F}(\delta;y,Q)/\partial\delta}\right]_{\delta=\delta(y,Q)}>0.$$

Together with the second-order condition and assumption 3, this ensures that donation sensitivity is increasing in income:

$$\frac{\partial^2\delta(Q,y)}{\partial y\partial Q}=\frac{\partial}{\partial y}\left[\frac{\partial MRS(y-\delta-\mathcal{T}(\delta),\delta;Q)/\partial Q}{-\partial\mathcal{F}(\delta;y,Q)/\partial\delta}\right]_{\delta=\delta(y,Q)}+\frac{\partial\delta(y,Q)}{\partial y}\frac{\partial}{\partial \delta}\left[\frac{\partial MRS(y-\delta-\mathcal{T}(\delta),\delta;Q)/\partial Q}{-\partial\mathcal{F}(\delta;y,Q)/\partial\delta}\right]_{\delta=\delta(y,Q)}>0.$$

In the numerical simulations presented in section 3.4, I impose CES function form assumptions on the utility function ϕ. Assumptions 1–4 are all satisfied in this case.

Equilibrium Policy

Let us assume that the equilibrium is convergent. The equilibrium tax rate $t_y^\star$ is then characterized by the candidate first-order condition

$$\int \gamma_\theta\cdot(\bar{y}-y_\theta)\,v'\left(y_\theta+t_y^\star(\bar{y}-y_\theta)\right)\mu(\mathrm{d}\theta)=0. \tag{26}$$

The weights γ_θ in this condition are as defined in equation (10). The other term in the integrand of equation (26) is the marginal utility of increasing the tax rate for type θ agents is

$$(\bar{y}-y_\theta)\,v'\left(y_\theta+t_y^\star(\bar{y}-y_\theta)\right).$$

Note, this marginal utility is negative for any citizen with above average income (those who lose from redistribution) and positive for those with below average income (who gain from redistribution). Equation (26) simply characterizes how political candidates weight the costs and benefits to different types of citizens when contemplating a change to the tax rate.

Equation (26) can alternatively be written in the form

$$\int(\bar{y}-y_\theta)\,v'\left(y_\theta+t_y^\star(\bar{y}-y_\theta)\right)\mu(\mathrm{d}\theta)=-\mathrm{Cov}\left(\frac{\gamma_\theta}{\bar{\gamma}},\ (\bar{y}-y_\theta)\,v'\left(y_\theta+t_y^\star(\bar{y}-y_\theta)\right)\right).$$

This formulation is useful because if the covariance term on the righthand side is equal to zero, then the equilibrium tax rate is the utilitarian-optimal rate $t_y^U = 1$.[18] The equilibrium tax rate $t_y^\star$ falls below this level whenever

$$\operatorname{Cov}\left(\frac{\gamma_\theta}{\bar{\gamma}},\ (\bar{y} - y_\theta)\, v'\left(y_\theta + t_y^\star (\bar{y} - y_\theta)\right)\right) < 0. \tag{27}$$

Suppose that the marginal utility of increasing the tax rate falls monotonically with income (this holds as long as curvature of the utility function $v(\cdot)$ is not too great).[19] Then condition (27) will be satisfied whenever higher income citizens have a relatively high political influence.

As discussed above, assumptions 1–4 imply that donation sensitivity is increasing in income ($\frac{\partial^2 \delta(Q,y)}{\partial y \partial Q} > 0$). This means that citizen types with higher incomes y_θ have a higher aggregate donation sensitivity $\bar{\delta}_\theta'(0)$. Recalling that the political influence of type θ citizens is

$$\gamma_\theta \equiv 2\eta_\theta(D)\, g(0) + \left[\int \eta_\theta'(D)\, \mu(\mathrm{d}\theta)\right] \bar{\delta}_\theta'(0),$$

this implies—all else equal—that higher income citizens have higher political influence. Thus, as long as citizen income is not too strongly negatively correlated with voter turnout rates, condition (27) will be satisfied and the equilibrium tax rate falls below the utilitarian rate: $t_y^\star < t_y^U$. Empirically, voter turnout rates are positively correlated with with income in many modern democracies.

Political Externality

Adopting the definition of political externality introduced in section 2, the political externality associated with the equilibrium tax rate $t_y^\star$ is

$$PE\left(t_y^\star\right) \equiv v(\bar{y}) - \int v\left(y_\theta + t_y^\star (\bar{y} - y_\theta)\right) \mu(\mathrm{d}\theta) \tag{28}$$

because under the welfare-maximizing tax rate of $t_y^U = 1$, all citizens have the average income of $\bar{y}$.

The political externality wedge for type θ citizens in this society is

$$\tau_\theta^{PE} = \left[\frac{\int (\bar{y} - y_\theta)\, v'\left(y_\theta + t_y^\star (\bar{y} - y_\theta)\right) \mu(\mathrm{d}\theta)}{\int \gamma_\theta \cdot (\bar{y} - y_\theta)^2\, v''\left(y_\theta + t_y^\star (\bar{y} - y_\theta)\right) \mu(\mathrm{d}\theta)}\right] (\bar{y} - y_\theta)\, v'\left(y_\theta + t_y^\star (\bar{y} - y_\theta)\right). \tag{29}$$

[18] In the absence of a behavioral response to taxation, fully confiscatory taxation is utilitarian-optimal.

[19] It can be shown that if the coefficient of relative risk aversion is less than one, marginal utility of increasing the tax rate is strictly decreasing in income:

$$-\frac{(y_\theta + t(\bar{y} - y_\theta))\, v''(y_\theta + t(\bar{y} - y_\theta))}{v'(y_\theta + t(\bar{y} - y_\theta))} < 1 \implies \frac{\mathrm{d}}{\mathrm{d}y_\theta}\left\{(\bar{y} - y_\theta)\, v'(y_\theta + t(\bar{y} - y_\theta))\right\} < 0.$$

3.2 Effects of a Donation Tax Reform

Reforms to the donation tax schedule impact citizen donation demand through the usual substitution and income effects of taxation. All else equal, increasing the marginal tax rate of some citizen leads to a reduction in their donations, the magnitude of which is measured by the *compensated elasticity of donations*:

$$\begin{aligned}\varepsilon(y,Q) &\equiv -\frac{1+\mathcal{T}'(\delta(y,Q))}{\delta(y,Q)}\left.\frac{\partial\delta(y,Q)}{\partial\mathcal{T}'}\right|^{C} \\ &= \frac{1+\mathcal{T}'(\delta(y,Q))}{\delta(y,Q)}\left[-\frac{\partial\mathcal{F}(\delta(y,Q);y,Q)}{\partial\delta}\right]^{-1} > 0.\end{aligned} \tag{30}$$

Increasing the average tax rate of some citizen—all else equal—also results in a reduction in donations due to the *income effect* of taxation

$$\frac{\partial\delta(y,Q)}{\partial\mathcal{T}} = -\frac{\partial\delta(y,Q)}{\partial y} < 0. \tag{31}$$

These standard behavioral responses—defined more precisely in appendix A—are components of the *fiscal externality* of a donation tax reform. Consider a tax reform which infinitesimally increases the marginal tax rate at some income level δ while holding rates constant elsewhere. The fiscal externality of such a reform is

$$\underbrace{-\left(\frac{\mathcal{T}'(\delta)}{1+\mathcal{T}'(\delta)}\right)\delta\bar{\varepsilon}(\delta)\,h(\delta)}_{\text{substitution effect at }\delta}\underbrace{-\int_{\delta}^{\infty}\mathcal{T}'(s)\,\mathbb{E}\left[\frac{\partial\delta(y_\theta,\xi)}{\partial y}|\delta(y_\theta,\xi)=s\right]h(s)\,\mathrm{d}s}_{\text{income effect above }\delta}. \tag{32}$$

The reform not only decreases donations at δ due to the substitution effect, it also increases the average tax rate of all citizens above δ, further reduces donations through incomes effects. The decreases in donations in turn lead to decreases in tax revenue: a fiscal externality.

The loss of income for the citizens above δ also leads to a welfare loss for these citizens. The average change in private welfare as a result of the reform is

$$-\int_{\delta}^{\infty}\mathbb{E}\left[{}^{\partial\phi(y_\theta,\xi)}/_{\partial y}|\delta(y_\theta,\xi)=\delta\right]h(s)\,\mathrm{d}s. \tag{33}$$

Determining the optimal donation tax also requires us to consider how a tax reform will impact the political externality. In section 2.3, we saw that reforms which impact donation behavior—such as donation taxes/subsidies—alter political influence through two mechanisms: the sensitivity effect and the campaign spending effect (see equation (18)). The campaign spending effect of a tax reform captures how the reforms alters political influence by changing the amount of money flowing to political campaigns. Because this effect depends only on how taxation impacts total donations, it can described using the standard behavioral responses to taxation described by equations (30) and (31).

Let us once again consider a tax reform which infinitesimally increases the marginal tax rate at some income level δ while holding rates constant elsewhere. The impact of this reform on the political externality via the campaign spending effect is

$$\frac{1}{2}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]\left(\frac{\delta\bar{\varepsilon}\left(\delta\right)h\left(\delta\right)}{1+\mathcal{T}'\left(\delta\right)}+\int_{\delta}^{\infty}\mathbb{E}\left[\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial y}|\delta\left(y_{\theta},\xi\right)=s\right]h\left(s\right)\mathrm{d}s\right). \tag{34}$$

As discussed above, the reform leads to reductions in donations through both substitution and income effects. This results in a reduction of total campaign spending per candidate. As discussed in section 2.3, to obtain the impact of this reduction on the political externality we simply multiply the total reduction of donations by $-\frac{1}{2}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]$.[20] Whether this results in an increase or a reduction of the political externality depends on whether the marginal dollar of campaign spending exacerbates or alleviates inequality of political influence among citizens.[21]

The sensitivity effect of a tax reform is more complex. The sensitivity effect of a tax reform depends on how the reform changes the importance of different citizens as a marginal source of funding for political campaigns. That is to say, the sensitivity effect of a reforms depends on how the reform changes the responsiveness of citizen donations to changes in candidate policy positions. Let us once again consider a tax reform which infinitesimally increases the marginal tax rate at some income level δ while holding rates constant elsewhere. The impact of this reform on the political externality via the sensitivity effect is

$$\underbrace{-\psi\int_{\delta}^{\infty}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial^{2}\delta\left(y,\xi\right)}{\partial Q\partial y}|\delta\left(y_{\theta},\xi\right)=\delta\right]h\left(s\right)\mathrm{d}s}_{\text{income sensitivity effect}}\underbrace{-\psi\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial Q\partial\mathcal{T}'}|\delta\left(y_{\theta},\xi\right)=\delta\right]h\left(\delta\right)}_{\text{price sensitivity effect}}$$
$$+\underbrace{\psi\frac{\mathrm{d}}{\mathrm{d}\delta}\left\{\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial Q\partial\mathcal{T}''}|\delta\left(y_{\theta},\xi\right)=\delta\right]h\left(\delta\right)\right\}}_{\text{convexity sensitivity effect}} \tag{35}$$

Equation (35) decomposes the sensitivity effect of the reform into three parts. I shall discuss each in turn.

1. **Income Sensitivity Effect:** As discussed above, this reform decreases the incomes of all citizens above δ. The *income sensitivity effect* captures the resulting reduction in the donation sensitivity of these citizens. That this effect reduces donation sensitivity follows from the fact that donation sensitivity is

[20]Note, we must multiply the effect by $-\frac{1}{2}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]$ rather than simply $-\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]$ because total donations are split evenly between the two candidates. Thus, reducing total donations by \$1 leads to a reduction of \$0.50 in campaign spending per candidate.

[21]Notice, describing the campaign spending effect of a tax reform is similar to analyzing the impact of a tax reform on a traditional atmospheric externality, where obtaining the externality effect of a tax reform consists of simply multiplying the behavioral response to the reform by a constant marginal social cost parameter.

increasing in income (assumption 4).

2. **Price Sensitivity Effect:** By increasing the marginal tax rate at δ, the tax reform increases the after-tax price of donating to a citizen's preferred political party. Intuitively, it seems reasonable to expect that make donations more expensive will make citizens' donation behavior less responsive to policy concessions at the margin. In appendix A, I show that this is indeed the case under assumption 4.

3. **Convexity Sensitivity Effect:** This is the least intuitive component of the sensitivity effect. It captures the implications of the fact that a local increase in the marginal tax rate at δ also changes the local convexity properties of the donation tax schedule (that is, the second derivative of the tax function $\mathcal{T}''(\delta)$). Roughly speaking, the reform increases the convexity of the tax schedule just below δ and decreases the convexity of the tax schedule just above δ. These changes in convexity in turn impact donation sensitivity, because—all else equal—a citizen facing a more convex tax schedule has less responsive donation behavior: that is to say $\frac{\partial\delta(y_\theta,\xi)}{\partial Q\partial\mathcal{T}''} < 0$. Intuitively, this is because a citizen wishing to donate additional funds in response to an increase in the perceived quality of their preferred candidate will increase their donation by a smaller amount if the marginal cost of donating increases quite steeply above their current amount.[22]

   To better understand how the convexity sensitivity effect works, consider the case where $\mathbb{E}\left[\tau_\theta^{PE}\frac{\partial\delta(y_\theta,\xi)}{\partial Q\partial\mathcal{T}''}|\delta(y_\theta,\xi)=\delta\right]$ is constant in δ, so that

$$\frac{\mathrm{d}}{\mathrm{d}\delta}\left\{\mathbb{E}\left[\tau_\theta^{PE}\frac{\partial\delta(y_\theta,\xi)}{\partial Q\partial\mathcal{T}''}|\delta(y_\theta,\xi)=\delta\right]h(\delta)\right\}=\mathbb{E}\left[\tau_\theta^{PE}\frac{\partial\delta(y_\theta,\xi)}{\partial Q\partial\mathcal{T}''}|\delta(y_\theta,\xi)=\delta\right]\cdot h'(\delta).$$

   Note, if $h'(\delta)<0$, this means that there is a greater mass of citizens just below δ than there are just below δ. Thus, if the marginal tax rate is increased at δ the group of citizens which sees the convexity of their tax schedule increase (those just below δ) is larger than the group which sees it decrease (those just above δ). Thus—when $\mathbb{E}\left[\tau_\theta^{PE}\frac{\partial\delta(y_\theta,\xi)}{\partial Q\partial\mathcal{T}''}|\delta(y_\theta,\xi)=\delta\right]$ is constant—the convexity effect measures the political externality impact of increasing the local convexity of the of the tax schedule if the donation density is locally decreasing, but measures the political externality impact of decreasing the local convexity of the of the tax schedule if the donation density is locally increasing.

In discussing each part of the sensitivity effect I have described how the tax reform should be expected to change donation sensitivity, but not how these changes translate into changes in the political externality. This is a complex matter, as each of these three effects changes the donation sensitivity of many different types

[22]See appendix A for a formal derivation of this fact.

of citizens simultaneously. Translating these changes into a political externality effect requires interacting the sensitivity impact for each group with the relevant group-specific political externality wedge τ_θ^{PE} and averaging across all impacted groups.

In the context of our example, recall that higher income citizens have higher donations, donation sensitivity, and political influence. In this circumstance, we should expect to see higher income citizens over-represented amongst higher donation quantities. In general, this sorting of citizen types across the donation distribution implies that the sensitivity mechanism can be leveraged to reduce inequality of political influence. For example, a pattern of increasing marginal rates may succeed in disproportionately reducing donation sensitivity at higher levels of donations through the price sensitivity effect while simultaneously reducing donation sensitivity through the convexity sensitivity effect. In section 3.4, I explore a numerical example where the optimal schedule has this property.

3.3 The Optimal Donation Tax/Subsidy Schedule

Now that we have discussed all the relevant effects that donation tax reforms have on tax revenue, private utility, and the political externality, we are ready to characterize the optimal nonlinear tax schedule. Here, I once again rely on the two period model of politics first introduced in section

Social Planner's Problem

I consider the problem of a social planner who is given a fixed budget of S (or a revenue target of $-S$) with which to finance the costs of a donation tax/subsidy program. Given this budget constraint, the planner seeks to find the tax schedule that seeks to minimize the political externality, while also accounting for the impact that a donation tax/subsidy schedule has on the period 1 utility of citizens.

Specifically, let period 1 indirect utility be

$$V_\theta^1(\xi) \equiv \phi\left(y_\theta - \delta - \mathcal{T}(\delta),\, \delta;\, \xi\right),$$

and period 2 indirect utility be

$$V_\theta^2(t_y) \equiv v\left(y_\theta + t_y\left(\bar{y} - y_\theta\right)\right),$$

so that the political externality is

$$PE(\mathcal{T}) \equiv \int V_\theta^2\left(t_y^U\right)\mu(\mathrm{d}\theta) - \int V_\theta^2\left(t_y^\star(\mathcal{T})\right)\mu(\mathrm{d}\theta)$$

where $t_y^\star(\mathcal{T})$ is the equilibrium tax rate that results under a given donation tax schedule. The social planner's problem is

$$\max_{\mathcal{T}}\left\{\int\left[\int_0^\infty V_\theta^1(\xi)\,\mathrm{d}\xi\right]\mu(\mathrm{d}\theta)-\rho\cdot PE(\mathcal{T})\right\} \tag{36}$$

subject to the budget constraint

$$S+2\int\left[\int_0^\infty \mathcal{T}(\delta_\theta(\xi))\,\mathrm{d}\xi\right]\mu(\mathrm{d}\theta)=0 \tag{37}$$

and a requirement that $\mathcal{T}(0)=0$.[23] The parameter ρ measures the weight the planner places on reducing the political externality relative to any period 1 utility costs of the program. As $\rho\to\infty$, the planner disregards any such costs, focusing exclusively on minimizing the political externality.

Notice, because we adopted a welfarist definition of political externality in section 2.1, the social planner's objective is equivalent to maximizing the weighted sum of average utility across both periods:

$$\max_{\mathcal{T}}\left\{\int\left[\int_0^\infty V_\theta^1(\xi)\,\mathrm{d}\xi\right]\mu(\mathrm{d}\theta)+\rho\cdot\int V_\theta^2\left(t_y^\star(\mathcal{T})\right)\mu(\mathrm{d}\theta)\right\}.$$

The formulation of the problem in equation (36) is helpful because it allows for easy substitution of alternative definitions of political externality, as I show in section 4.

Characterizing the Optimal Tax Schedule

For a heuristic derivation of the optimal tax schedule, note that in section **??**, we discussed all the ways that a specific donation tax reform impacts the social planner's problem. In particular, we considered a reform that infinitesimally increases the marginal tax rate at δ while holding marginal rates constant everywhere else. The effect of the reform on the planner's budget constraint (equation 37) is presented in equation (32). Equations (33), (34), and (35) capture the effects of the reform on the planner's objective function (equation 36).

Letting λ be the multiplier on the planner's budget constraint, we can obtain the total effect of this reform on welfare by summing all four equations together, with impacts on the planner's objective function discounted by $\frac{1}{\lambda}$ and any political externality effects weighted by ρ. This leads to the following welfare effect of the reform

[23] This last requirement substantially restricts the planner's ability to use the donation tax/subsidy program as a mechanism for period 1 redistribution. The planner still has some incentive to do this as long as $\rho<\infty$, but cannot do so very effectively.

$$\int_{\delta}^{\infty}\left(1-\bar{\omega}\left(s\right)\right)h\left(s\right)\mathrm{d}s-\left(\frac{\mathcal{T}'\left(\delta\right)-\frac{\rho}{2\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]}{1+\mathcal{T}'\left(\delta\right)}\right)\delta\bar{\varepsilon}\left(\delta\right)h\left(\delta\right)-\frac{\rho\psi}{\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial Q\partial\mathcal{T}'}|\delta\left(y_{\theta},\xi\right)=\delta\right]h\left(\delta\right)$$
$$+\frac{\rho\psi}{\lambda}\frac{\mathrm{d}}{\mathrm{d}\delta}\left\{\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial Q\partial\mathcal{T}''}|\delta\left(y_{\theta},\xi\right)=\delta\right]h\left(\delta\right)\right\} \quad (38)$$

where the average social marginal utility of income for citizens with donations δ is

$$\bar{\omega}\left(\delta\right)\equiv\frac{\mathbb{E}\left[\partial\phi(y_{\theta},\xi)/\partial y|\delta\left(y_{\theta},\xi\right)=\delta\right]}{\lambda}+\left(\mathcal{T}'\left(\delta\right)-\frac{\rho}{2\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]\right)\cdot\mathbb{E}\left[\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial y}|\delta\left(y_{\theta},\xi\right)=\delta\right]$$
$$+\frac{\rho\psi}{\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial^{2}\delta\left(y_{\theta},\xi\right)}{\partial Q\partial y}|\delta\left(y_{\theta},\xi\right)=\delta\right]. \quad (39)$$

For any such reform, equation (38) should be equal to zero under a (locally) optimal tax schedule, so that the planner does not wish to deviate from the schedule. Thus, we obtain the following necessary condition characterizing the optimal donation tax schedule:

$$\frac{\mathcal{T}'\left(\delta\right)-\overbrace{\frac{\rho}{2\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]}^{\text{spending effect}}}{1+\mathcal{T}'\left(\delta\right)}=\underbrace{\frac{1}{\bar{\varepsilon}\left(\delta\right)}\cdot\left(\frac{1-H\left(\delta\right)}{\delta h\left(\delta\right)}\right)\cdot\left(1-\mathbb{E}\left[\bar{\omega}\left(s\right)|s>\delta\right]\right)}_{\text{standard ABC formula}}$$
$$\underbrace{+\frac{1}{\delta}\cdot\frac{\rho\psi}{\lambda}\cdot\left(\overbrace{-\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial Q\partial\mathcal{T}'}|\delta\left(y_{\theta},\xi\right)=\delta\right]}^{\text{price sensitivity effect}}+\overbrace{\frac{1}{h\left(\delta\right)}\frac{\mathrm{d}}{\mathrm{d}\delta}\left\{\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta\left(y_{\theta},\xi\right)}{\partial Q\partial\mathcal{T}''}|\delta\left(y_{\theta},\xi\right)=\delta\right]h\left(\delta\right)\right\}}^{\text{convexity sensitivity effect}}\right)}_{\text{price and convexity sensitivity effects}}$$
$$(40)$$

This necessary condition is derived more formally in appendix A via the calculus of variations.[24]

Equation (40) parallels the standard ABC formula that is ubiquitous in characterizations of optimal nonlinear tax schedules but it is augmented by two additional terms that account for political externality effects of taxation. First, the campaign spending effect $\frac{\rho}{2\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]$ is subtracted from the numerator of the lefthand side of the equation. To better understand the role of this effect suppose that $\psi=0$, so that the sensitivity effects drop out of equation (40). In this case, the optimal tax treatment of donations follows the Pigouvian prescription for dealing with externalities: in the presence of an externality, the tax rate on an externality-generating good should be increased by the marginal social cost of the externality (or decreased by its marginal social benefit). In the case of political externality effects which occur through the campaign

[24] I do not derive sufficient conditions for optimality in this paper.

spending mechanism, the marginal social cost/benefit of campaign spending is $\frac{\rho}{2\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]$.

Now we turn our attention to the sensitivity effects appearing on the lefthand side of equation (40). To understand the role of these effects, consider a case like that described in section 3.2 where at some value of δ, $\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta(y_{\theta},\xi)}{\partial Q\partial T''}|\delta\left(y_{\theta},\xi\right)=\delta\right]$ is locally constant, so that the convexity sensitivity effect simplifies to $\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta(y_{\theta},\xi)}{\partial Q\partial T''}|\delta\left(y_{\theta},\xi\right)=\delta\right]h'\left(\delta\right)$. Further suppose that the density of donations is decreasing at δ (i.e. $h'\left(\delta\right)<0$). In this case, a higher tax rate at δ serves to decrease donation sensitivity among citizens at δ through both the price and convexity sensitivity effects. If $\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta(y_{\theta},\xi)}{\partial Q\partial T'}|\delta\left(y_{\theta},\xi\right)=\delta\right]<0$ and $\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta(y_{\theta},\xi)}{\partial Q\partial T''}|\delta\left(y_{\theta},\xi\right)=\delta\right]<0$, then such a change in donation sensitivity would serve to reduce the political externality. In this case the term on the righthand side of equation (40) labeled "price and convexity sensitivity effects" is positive, implying higher optimal marginal tax rates on donations at δ than would be prescribed absent these sensitivity effects.

Finally, it is important to keep in mind that political externality effects of taxation also augment the standard optimal nonlinear tax formula via their presence in the social marginal utility of income, as shown in equation (39).

3.4 Numerical Example

While equation (40) helps to shed some light on optimal donation taxation, it is a somewhat opaque characterization. To shed further light on optimal donation tax, here I explore some simple numerical simulations. While these simulations are not based on real world data, they provide a concrete illustration of some of the potential implications of the theory of optimal donation taxation presented above.

Setup

I assume that period 1 utility has the CES form

$$\phi\left(C,\delta;Q\right)\equiv\frac{1}{1-\sigma}\left(\left[C^{\frac{e-1}{e}}+Q^{\frac{1}{e}}\delta^{\frac{e-1}{e}}\right]^{\frac{e}{e-1}}\right)^{1-\sigma}$$

and that period 2 utility has the isoelastic form

$$v\left(C\right)\equiv\frac{C^{1-\sigma}}{1-\sigma}.$$

Notice, the concavity of the utility function in both periods is governed by the parameter σ. The elasticity of substitution between donations and other period 1 consumption is e. I assume that both σ and e are fixed across all citizens. In all my simulations, I assume that $\sigma = 0.8$ and $e = 0.7$.

I consider a population consisting of $N = 200$ citizen types, each of which has some income y_θ drawn from a standard log-normal distribution. Each individual citizen also has a preference shock ξ draw from a mean zero normal distribution with a standard deviation $b = 0.1$.

Finally, I adopt the following functional form for describing citizen turnout rates:

$$\eta_\theta (D) \equiv 1 - \exp \{-\beta_0 - \beta_y y_\theta - \beta_D D\} .$$

In every simulation, I assume that $\beta_0 = \beta_y = \beta_D = 0.1$. Notice, this functional form results in turnout rates which are increasing in income and in total campaign spending. It also implies that there are diminishing returns to campaign spending. As well, a marginal dollar of campaign spending increases voter turnout more for low income citizens than it does for high income citizens.

Baseline Political Equilibrium

Given the assumptions described above, let us consider what the political equilibrium looks like in the absence of donation taxation. Figure 1 shows how normalized political influence ($\frac{\gamma_\theta}{\mathbb{E}[\gamma_\theta]}$) varies as a function of the quantile of citizen income. Political influence is monotonically increasing in citizen income in this baseline political equilibrium. This inequality of political influence stems from two sources: inequality of voter turnout rates (figure 2) and inequality of donation sensitivity (figure 3). In the baseline equilibrium, each candidate spends an amount equal to 6.94% of total income, resulting in a marginal value of campaign spending of $\psi = 0.0388$. In the baseline equilibrium, the tax rate implemented in period 2 is $t_y^\star = 0.363$.

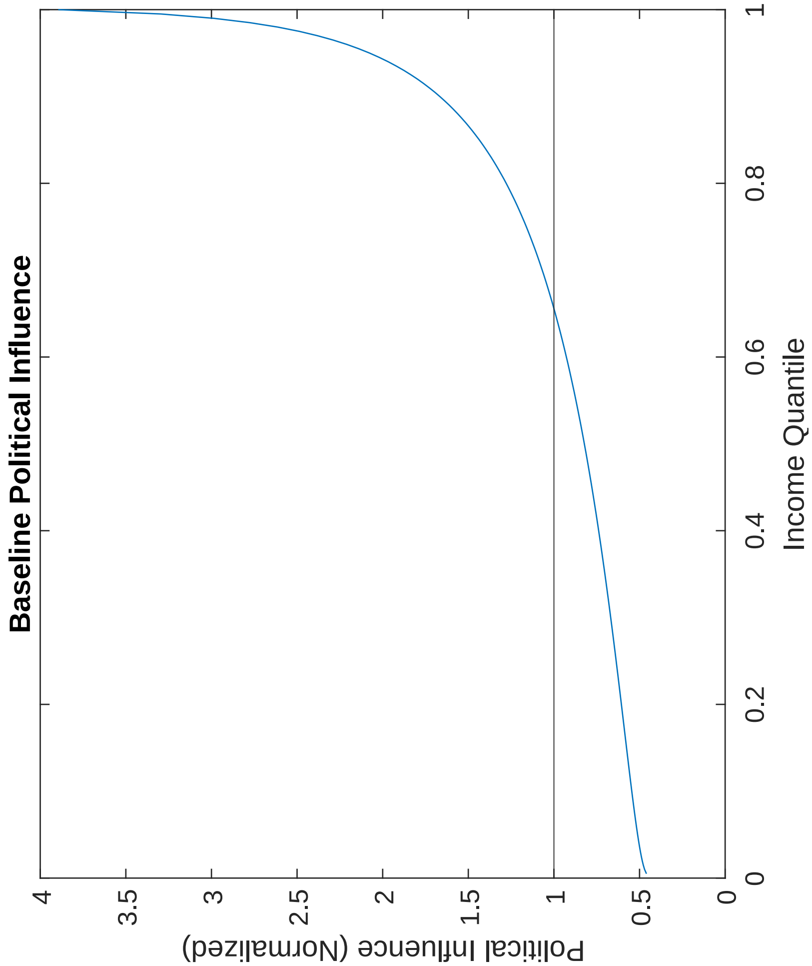

Figure 1: Baseline Political Influence

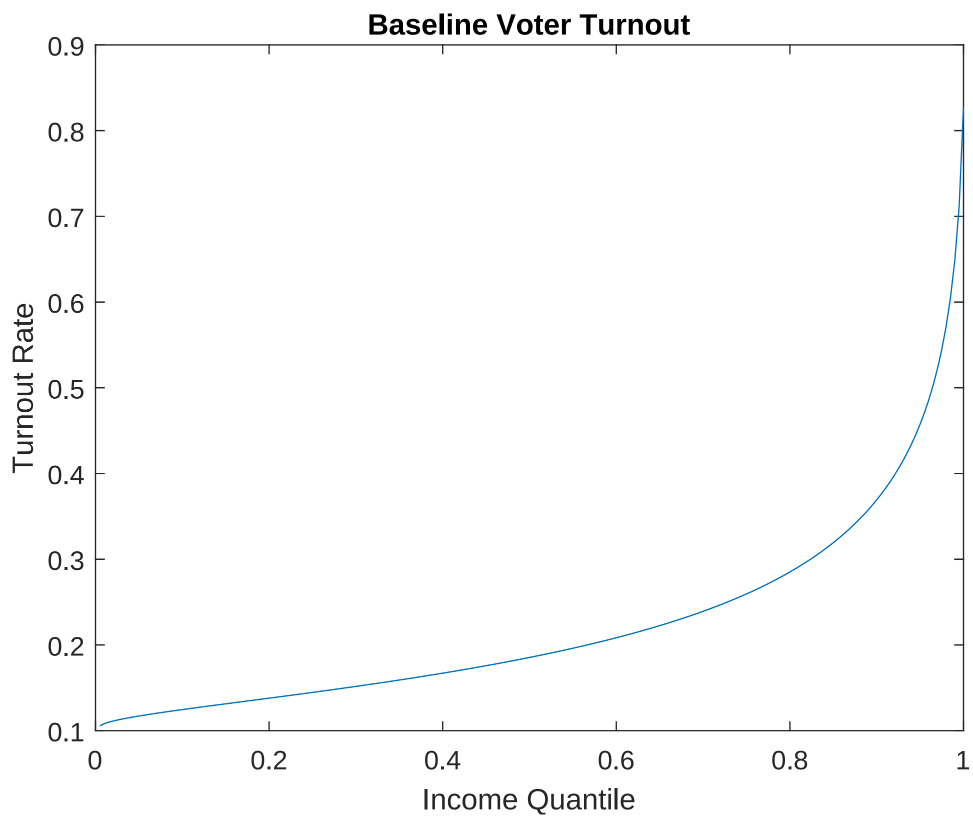

Figure 2: Baseline Turnout Rates

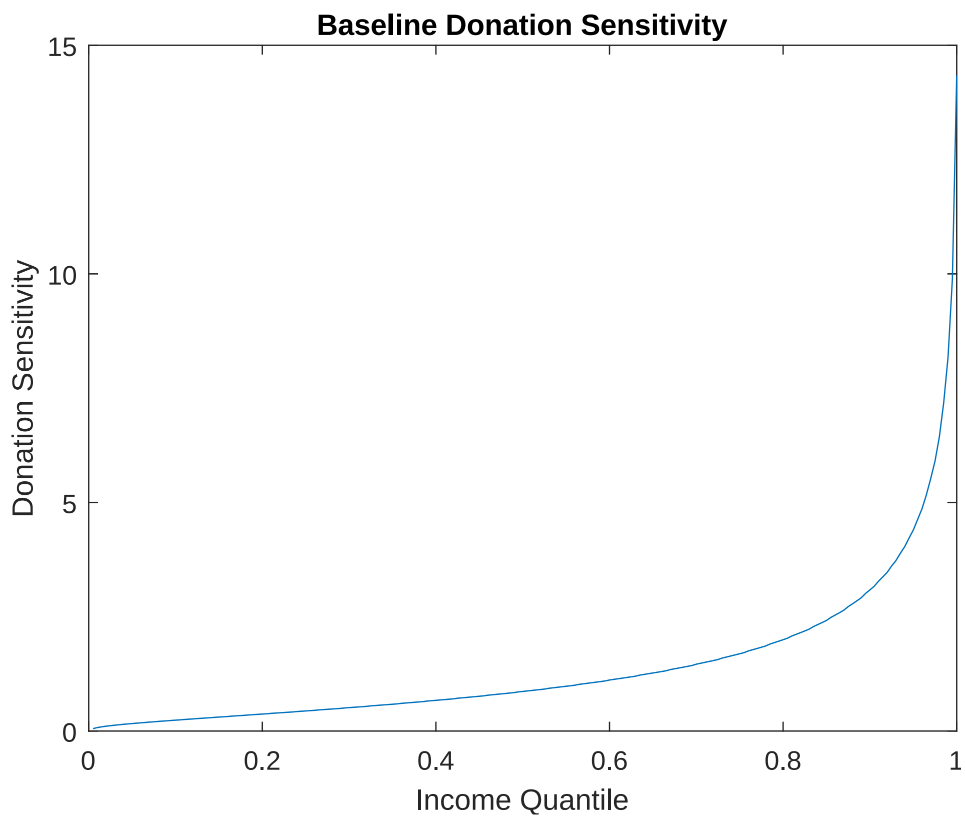

Figure 3: Baseline Donation Sensitivity

Optimal Tax Schedules

Now suppose that a social planner of the kind described in section 3.3 intervenes in this society, implementing the optimal tax schedule described by equation (40). The planner's weight on the political externality is assumed to be $\rho = 1$. I solve for the optimal tax/subsidy schedule in this society in different seven scenarios, varying the size of the budget that the planner has available for financing the costs of this program. In practice, I do this by solving for the optimal schedule given a fixed value of the multiplier λ on the planner's budget constraint. I obtain a discrete approximation to the optimal schedule for each value of

$$\lambda \in \{0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6\}.$$

This is accomplished using an iterative procedure which is described in appendix B.

The resulting optimal marginal rate schedules are presented in figure 4. In each case, the schedule features a -100% marginal tax rate on the first dollar of donations (i.e. a 100% marginal subsidy), effectively making the first dollar of donations free. However, the tax schedule is convex, so the marginal tax rate increases significantly at larger values of donations. At sufficiently high donation amounts, the marginal tax rate becomes positive, gradually growing to reach a rate of more than 200%, meaning that it will cost larger donors more than \$3 to donate \$1 to their preferred candidate.

While figure 4 appears to show that optimal tax rates level out at the largest donation amounts, this is a somewhat misleading perspective. For an alternative perspective on the optimal tax schedule, consider figure 5, which plots optimal marginal tax rates as a function of the quantile of donation amount. This figure shows that the majority of donors face a negative marginal tax rate on donations. Furthermore, the vast majority of donors face tax schedules feature substantial local convexity.

It is also important to highlight a fact which cannot be well-captured in these numerical simulations: the optimal top marginal rate in this example diverges to positive infinity. In appendix B, I show this formally. This implies that—rather than decreasing—the convexity of the optimal tax schedule actually reaches its most extreme at the top of the tax schedule.

In addition to exhibiting certain unifying features discussed above, figures 4 and 5 also shows how the optimal schedule varies with the social planner's budget. Note that the size of the social planner's budget is inversely related to the value of λ. Thus, these figures show that when the social planner's budget is more generous, they extend the range of donation amounts at which a negative marginal tax rate applies, consequently expanding the fraction of donors who face negative marginal rates.

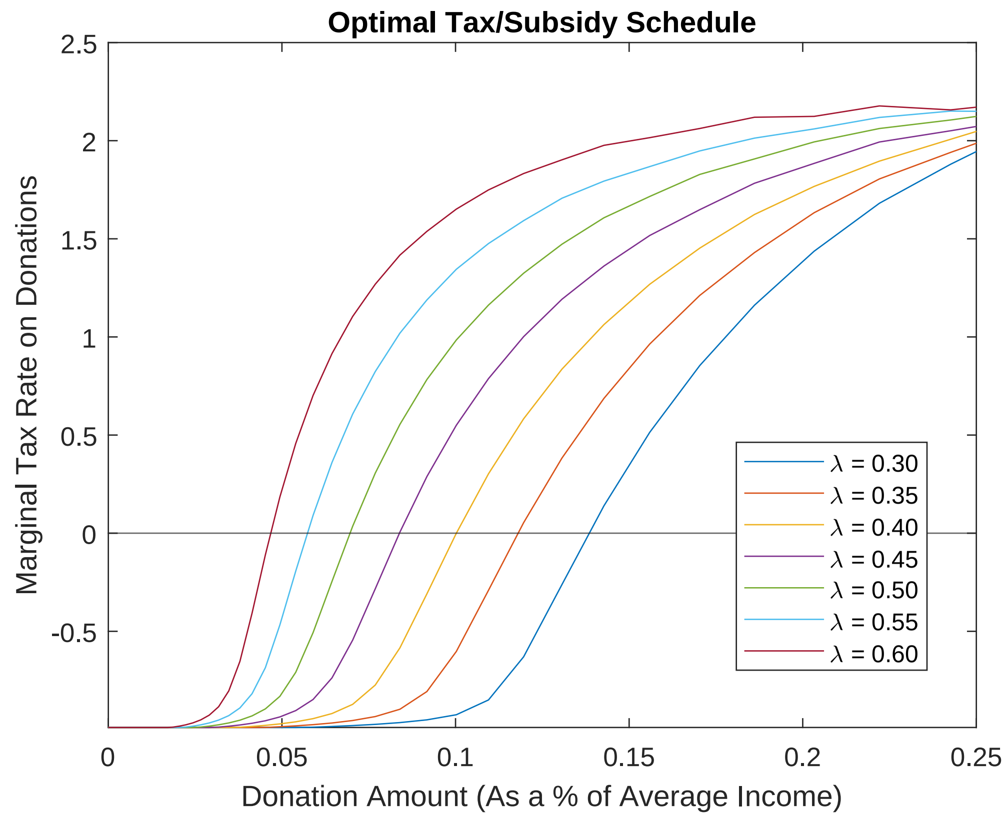

Figure 4: Optimal Schedules

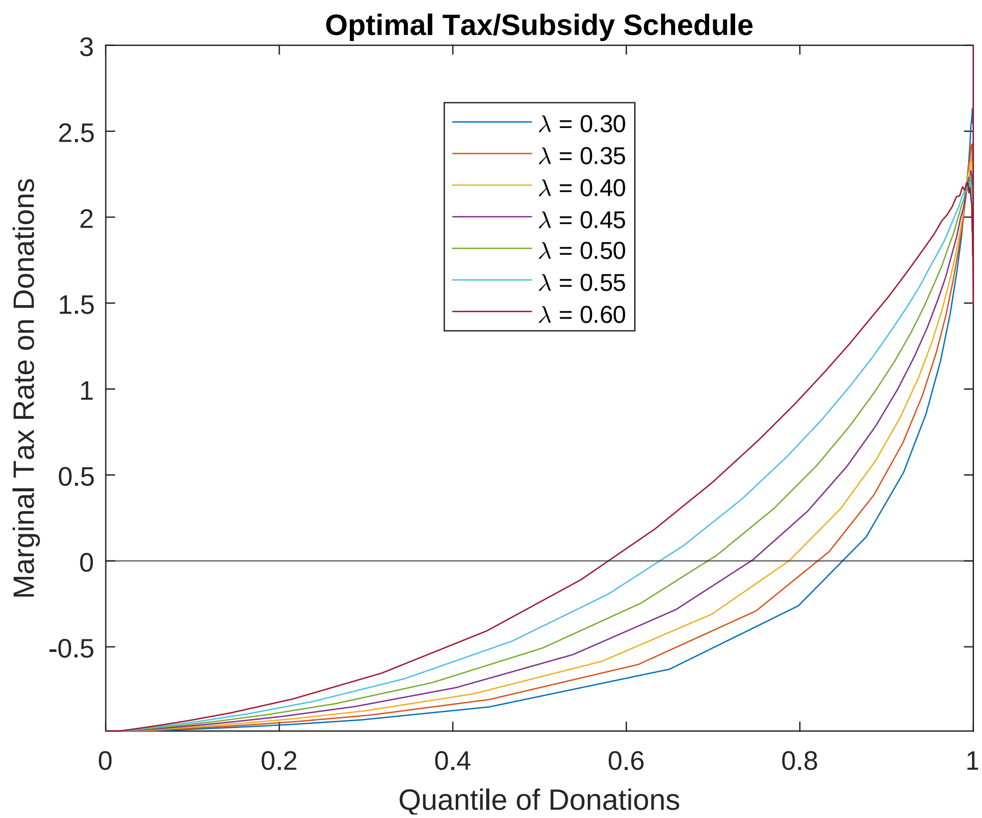

Figure 5: Optimal Schedules

Table 1 shows how each of my simulated optimal marginal tax schedules impact the political equilibrium. In each case, implementing the optimal donation tax schedule successfully alters the political equilibrium, inducing changes in political influence that lead political candidates to adopt a more redistributive policy platform in the election (i.e. a higher equilibrium tax rate $t_y^\star$). As would be expected, given a larger budget the planner is able to bring about a larger change in the equilibrium tax rate.

Figure 6 shows the specific changes political influence that result in each scenario. The optimal tax increases the normalized political influence of lower income citizen while decreasing it for higher income citizens, especially for those with the highest incomes. These changes can be further decomposed into changes in voter turnout rates (figure 7), changes in donation sensitivity (figure 8), and changes to the marginal value of campaign spending (table 1).

As figure 8 shows, in every case the adoption of an optimal donation tax schedule leads to substantial changes in donation sensitivity, with the lowest income citizens seeing very largest increases, matched by very large decreases among the highest income citizens. This is not surprising given the negative marginal tax rates applied at low donation amounts—where low income citizens are over-represented—and the high marginal tax rates applied at high levels of donations—where high income citizens are over-represented. The strong convexity of the tax schedules depicted in figures 4 and 5 likely also contributes to this result. Notice, while increasing in budget of the social planner increases the magnitude of these effects at the bottom of the income distribution, near the top of the income distribution is actually decreases the magnitude of the effect and—in some cases—flips the sign of the effects. That is, given a larger income, the planner prefers to increase the donation sensitivity of a larger set of citizens.

As figure 7 shows, the impact of the optimal tax schedule on voter turnout can differ substantially depending on the planner's budget. When the planner has a relatively smaller budget, voter turnout decreases for all citizens, especially low income citizens. By contrast, when the planner has a relatively large budget, voter turnout increases for all citizens, especially low income citizens. This is not surprising. As table 1 shows that the impact of the optimal tax schedule on total campaign spending differs depending on the planner's budget: increasing spending when the budget is higher, decreasing spending when the budget is lower. Given the assumed turnout function, voter turnout is increasing in campaign spending *and* this effect is larger for low income voters. Together, these two facts explain the pattern seen in figure 7.

This pattern in the impact of the optimal tax schedule on campaign spending also manifests in the impact on the marginal value of campaign spending (ψ). This parameter is decreased in high budget scenarios. Recall, a lower value of ψ results in a lower weight being placed on donation sensitivity in total political influence. This can lead to a decrease in inequality of political influence if inequality of donation sensitivity is greater

	Baseline	$\lambda = 0.30$	$\lambda = 0.35$	$\lambda = 0.40$	$\lambda = 0.45$	$\lambda = 0.50$	$\lambda = 0.55$	$\lambda = 0.60$
Equilibrium Tax Rate ($t_y^\star$)	0.3630	0.4029	0.3964	0.3912	0.3869	0.3833	0.3802	0.3775
Cost (as % of Total Income)	0	0.1897	0.1538	0.1239	0.0972	0.0738	0.0527	0.0327
Donations (as % of Total Income)	0.0694	0.1135	0.0989	0.0873	0.0773	0.0690	0.0619	0.0557
Marginal Value of Spending (ψ)	0.0388	0.0387	0.0387	0.0388	0.0388	0.0388	0.0388	0.0389

Table 1: Impacts of Optimal Tax Schedules

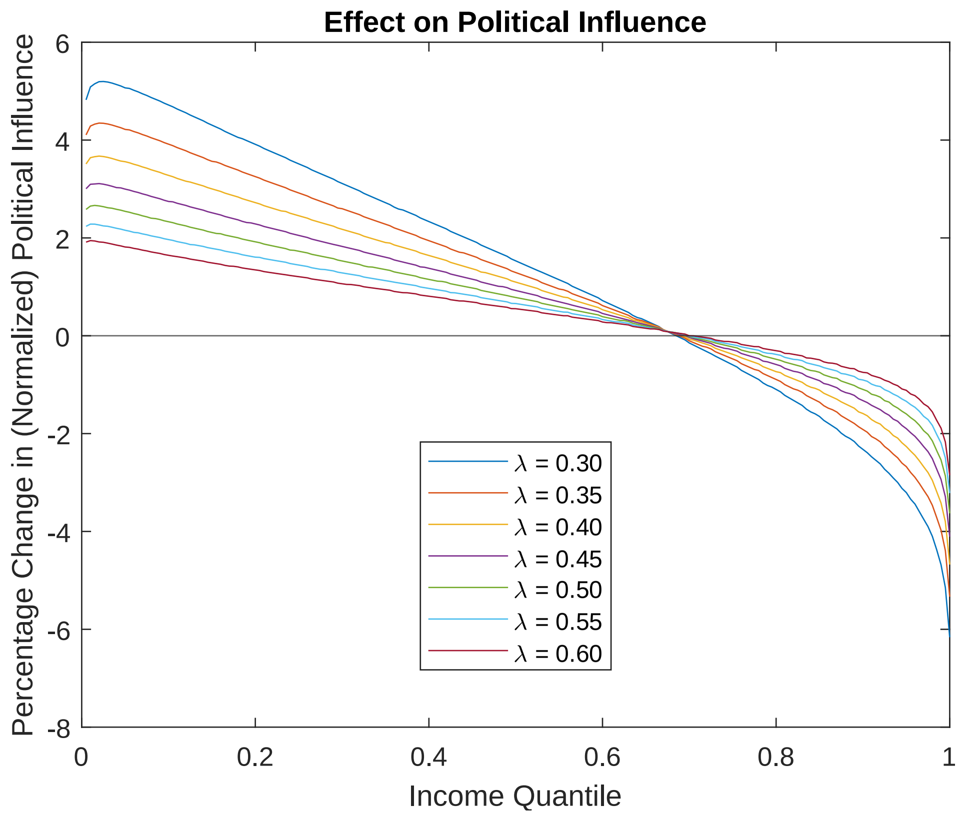

Figure 6: Effect on Political Influence

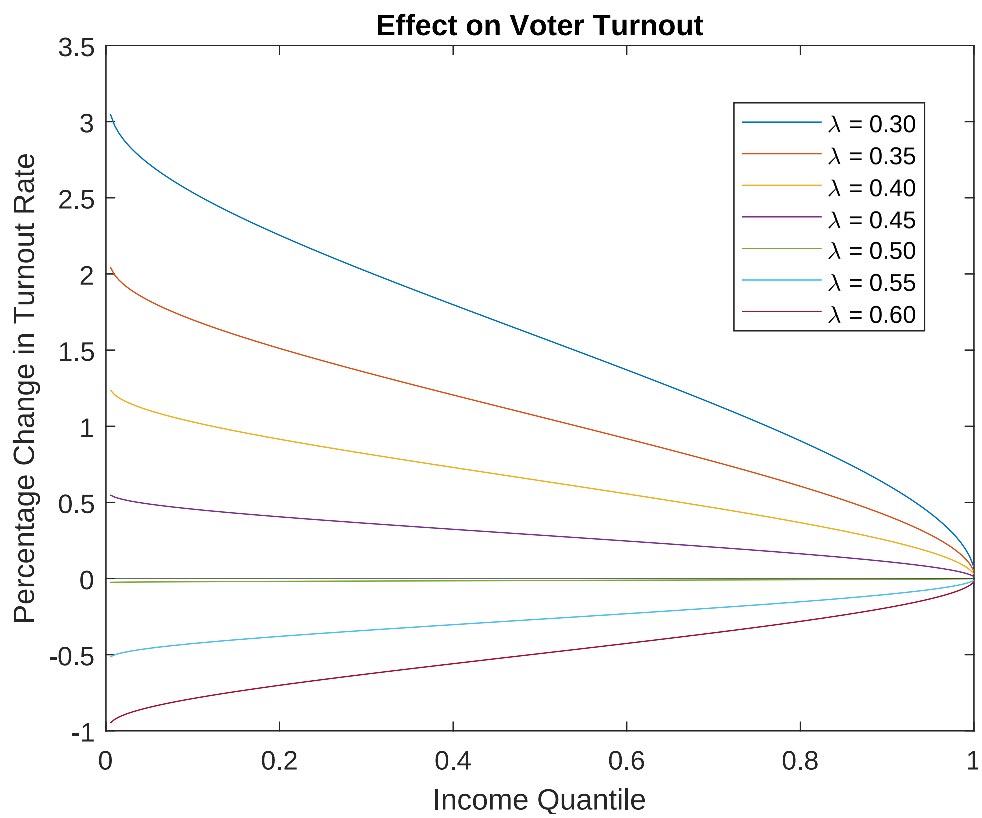

Figure 7: Effect on Turnout Rates

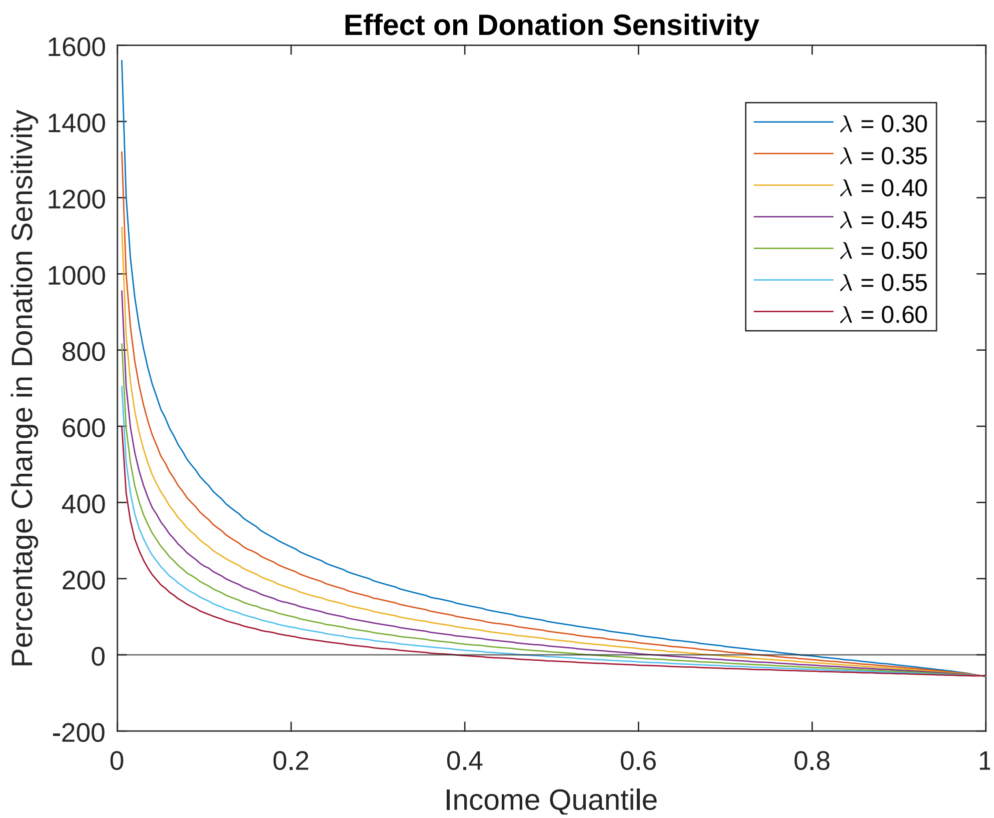

Figure 8: Effect on Donation Sensitivity

than inequality in turnout rates.

Discussion

While the results of this simulation exercise do not have any direct policy implications, they do show that applying a progressive subsidy schedule to political donations can be justified by a concern for *political externalities* as I have defined them in this paper. As I noted in the introduction, Canadian political contribution tax credits (at both the federal and provincial levels) offer real world examples of this kind of tax treatment of political donations.

Intriguingly, two Canadian territories actually do have tax credit schedules feature a -100% marginal tax rate on the first dollar of donations as in my simulated optimal tax schedules. As well, the Canadian regulatory regime features annual caps on the total donations an individual can make, which is arguably closely related to the infinite top marginal rate result present in my example. It would be valuable to explore under what conditions optimal donation tax schedules should be expected to exhibit these two properties, but I leave this task for future work.

4 Direct Aversion to Inequality of Political Influence

The definition of political externality presented in section 2.1 and subsequently applied in section 3 offers a welfarist perspective on the normative analysis of campaign finance. For reference, in this section I refer to this measure of the political externality as PE^{welf}. According to PE^{welf}, inequality of political influence is only objectionable inasmuch as it leads to a sub-optimal equilibrium policy choice. While perhaps intuitively appealing to economists, this perspective is not universally endorsed in the political finance literature. In this section, I discuss the implications an alternative, non-welfarist normative perspective on campaign finance issues.

A common normative stance in the political finance literature is the view that inequality of political influence is objectionable in and of itself, regardless of whether or not it leads to suboptimal policy choices. To better understand this viewpoint, consider a scenario where wealthy citizens have disproportionate influence in the political process but, coincidentally, these wealthy citizens have policy preferences such that the equilibrium policy outcome is the welfare-maximizing policy. According to PE^{welf}, there is no political externality associated with the disproportionate influence of the wealthy in this scenario. Overton [2004] labels this situation “virtual representation by the wealthy” and suggests that it is objectionable on the grounds that the less wealthy citizens' are having their "individual autonomy" undermined. More generally, political philosophers who consider the pursuit of a democratic ideal of "political equality" to be a good in and of itself [Dahl, 2006], might object to reliance on PE^{welf} as a normative criterion.

In this section, I will consider an alternative definition of the political externality which is better alligned with these perspectives:

$$PE^{ineq} \equiv - \int \log\left(\frac{\gamma_\theta}{\bar{\gamma}}\right) \mu\left(\mathrm{d}\theta\right).$$

According to this definition, the political externality associated with a given electoral equilibrium is the measured inequality in the political influence weights which characterize the equilibrium. Here, I have adopted a specific measure of inequality—called mean-log deviation—which has been frequently employed in the literature on income inequality. Other measures of inequality might also be appropriate for quantifying political inequality.

Notice, inequality of influence which due to inequality in donation sensitivity is not given any special status in this definition of the political externality (PE^{ineq}). Rather, a citizens' political influence reflects influence through both the voting mechanism and the donations mechanism. Arguably, this approach is consistent with the broad definitions of political (in)equality most commonly found in the literature [Dahl, 2006, Rosset, 2016, Shore, 2016].[25]

By construction, the use of PE^{ineq} as a political externality definition addresses normative objections to "virtual representation by the wealthy" situations. Of course, this necessarily comes at a cost: according to PE^{ineq}, a campaign finance reform which enhances aggregate welfare (i.e. decreases PE^{welf}), but which simultaneously increases political inequality (i.e. increases PE^{ineq}) is normatively objectionable. It is noteworthy, however, that both PE^{welf} and PE^{ineq} are minimized when there is perfect equality of political influence (i.e. if $\gamma_\theta = \gamma \in \mathbb{R}_+$ for all θ), because in this case the welfare maximizing policy will be adopted. Thus, in the limiting ideal case, these two normative criteria coincide.

It should be noted that this formalization of political equality as a normative objective in some ways falls short of the ideas that inspire it. For example, Dahl [2006] defines political equality by reference to a hypothetical ideal democracy in which citizens should not only have equal influence on the outcome, but also must have sufficient opportunities to engage in information acquisition and communicating with one another about their policy preferences. On the other hand, by adopting a simplified mathematical criteria for measuring political inequality, my criteria provides the precision necessary to offer clear answers to policy questions.

[25]While some scholars have argued for a distinction to be draw between inequality of influence caused by campaign finance and that due to other factors, it is not always clear that these arguments are built upon a claim these two mechanisms are normatively distinct. For example, Overton [2004] argues for such a distinction, based in part on a claims that other factors are less important to inequality of political influence and, furthermore, are harder for campaign finance reforms to address.

This trade-off is perhaps analogous to the compromises inherent in using a maximin social welfare function to approximate Rawlsian notions of distributive justice.

Application

It is quite straightforward to generalize the results of sections 2.1 and 3 to the case where PE^{ineq} is used in place of PE^{welf} as a working definition of political externality. Recall that the political externality wedge of type θ citizens—as introduced in section 2.3—measures the impact that increases the political influence of those citizens has on PE^{welf}. Consider an alternative definition of the political externality wedge of type θ citizens:

$$\tau_\theta^{PE} \equiv \frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}.$$

This new wedge measures the impact that increasing the political influence of type θ citizens has on PE^{ineq} (i.e. $\tau_\theta^{PE} = \frac{\mathrm{d}PE^{ineq}}{\mathrm{d}\gamma_\theta}$). We can characterize the optimal tax schedule in a case where PE^{ineq} is used in place of PE^{welf} by simply substituting this alternative wedge into the optimal tax formula presented in equation (40).

5 Conclusion

Can tax policy be used to facilitate the proper functioning of a modern democracy? At present, a tax economist faced with this question cannot presented a complete answer. To do so requires not only a commitment to a positive theory democratic governance which captures the most important institutional features at stake, but also a normative framework for interpreting the behavior of such a model as well as an appropriate normative thought experiment within which the question can be posed and answered. While this paper does not represent the final word on these issues, it does serve to demonstrate various difficulties associated with this task. At each stage, different choices might have been in developing the theory, which might yield distinct policy conclusions.

Yet, if the goal of optimal tax theory is to provide a guide to policy action, the omission of the topic of political externalities seems untenable. Tax policies ostensibly designed to address such externalities already exist. Where they do not yet exist, some campaign finance reformers, political finance scholars, and tax economists have called for their introduction. Further development of formal frameworks for evaluating such claims is by help to clarify the nature of disagreements about these issues and suggest future avenues for productive empirical work which can inform policy-making.

The paper necessarily leaves a great many questions about optimal taxation with political externalities unanswered. For example, although I have shown that a reasonably defined political externality will not necessarily be of the standard atmospheric kind, I have not resolved the question of whether some version of the principle of targeting applied to political finance transactions. If donation taxes are set optimally, under what circumstances will there still be scope for the social planner to adjust income or wealth taxes to achieve further corrective benefits? This is but one of many promising theoretical research questions I leave to future work.

This paper also fails to confront the daunting but potentially valuable task of applied welfare analysis. The optimal tax formula presented in section 3 contains novel sufficient statistics some of which appear—at least superficially—to require an amount of information well beyond what is available to policy makers. On the one hand, prior empirical work has certainly investigated questions about how voter turnout rates differ across groups and what the marginal effect of campaign spending is. On the other hand, the effect of taxation of donation sensitivity is an empirical parameter which—to my knowledge—has not been estimated in any prior literature. It is beyond the scope of this paper to collect empirical evidence on these novel sufficient statistics.

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A Optimal Tax Derivations

To obtain the optimal tax schedule, we will consider tax functions of the form

$$T(\delta;\kappa) \equiv \mathcal{T}(\delta) + \kappa\mathcal{R}(\delta)$$

where $\mathcal{R} : \mathbb{R}_+ \to \mathbb{R}$ is a *tax reform*, which is some twice continuously differentiable function satisfying $\mathcal{R}(0) = 0$. Note, if $\mathcal{T}$ is the optimal tax schedule, then for any tax reform $\mathcal{R}$, we will find that the total first-order welfare effect of the reform is zero.

Let period 1 indirect utility be

$$V_\theta^1(\xi) \equiv \phi\left(y_\theta - \delta - \mathcal{T}(\delta) - \kappa\mathcal{R}(\delta),\, \delta;\, \xi\right),$$

and period 2 indirect utility be

$$V_\theta^2(t_y) \equiv v\left(y_\theta + t_y\left(\bar{y} - y_\theta\right)\right).$$

The social planner's problem is

$$\max\left\{\int\left[\int_0^\infty V_\theta^1(\xi)\,\mathrm{d}\xi\right]\mu(\mathrm{d}\theta) - \rho\cdot PE\right\}$$

subject to the budget constraint

$$S = \int\left\{\int_0^\infty\left[\mathcal{T}(\delta_\theta(\xi)) + \kappa\mathcal{R}(\delta_\theta(\xi))\right]\mathrm{d}\xi\right\}\mu(\mathrm{d}\theta)$$

and a requirement that $\mathcal{T}(0) = 0$. The political externality is

$$PE \equiv \int V_\theta^2\left(t_y^U\right)\mu(\mathrm{d}\theta) - \int V_\theta^2\left(t_y^\star(\mathcal{T})\right)\mu(\mathrm{d}\theta).$$

For a given tax reform, the donation demand of a citizen with income y and quality $Q > 0$ satisfies the first-order condition

$$1 + \mathcal{T}'(\delta(y,Q)) + \kappa\mathcal{R}'(\delta(y,Q)) - MRS\left(y - \delta(y,Q) - \mathcal{T}(\delta(y,Q)) - \kappa\mathcal{R}(\delta(y,Q)),\, \delta(y,Q);\, Q\right) = 0.$$

Assuming that assumptions 1–4 hold for any reformed tax schedule $T(\delta;\kappa)$ under consideration, we have the following behavioral response to a local reform of the tax schedule

$$\left.\frac{\mathrm{d}\delta\left(y,Q\right)}{\mathrm{d}\kappa}\right|_{\kappa=0}=\mathcal{R}'\left(\delta\left(y,Q\right)\right)\left.\frac{\partial\delta\left(y,Q\right)}{\partial\mathcal{T}'}\right|^{C}+\mathcal{R}\left(\delta\left(y,Q\right)\right)\frac{\partial\delta\left(y,Q\right)}{\partial\mathcal{T}}$$

where

$$\begin{aligned}\left.\frac{\partial\delta\left(y,Q\right)}{\partial\mathcal{T}'}\right|^{C}&\equiv\left.\frac{\mathrm{d}\delta\left(y,Q\right)}{\mathcal{R}'\left(\delta\left(y,Q\right)\right)\mathrm{d}\kappa}\right|_{\kappa=\mathcal{R}(\delta(y,Q))=0}\\&=-\frac{1}{\mathcal{T}''\left(\delta\left(y,Q\right)\right)+\left(1+\mathcal{T}'\left(\delta\left(y,Q\right)\right)\right){}^{\partial MRS(y,Q)}/_{\partial C}-{}^{\partial MRS(y,Q)}/_{\partial\delta}}<0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\delta\left(y,Q\right)}{\partial\mathcal{T}}&=\left.\frac{\mathrm{d}\delta\left(y,Q\right)}{\mathcal{R}\left(\delta\left(y,Q\right)\right)\mathrm{d}\kappa}\right|_{\kappa=\mathcal{R}'(\delta(y,Q))=0}\\&=-\frac{{}^{\partial MRS(y,Q)}/_{\partial C}}{\mathcal{T}''\left(\delta\left(y,Q\right)\right)+\left(1+\mathcal{T}'\left(\delta\left(y,Q\right)\right)\right){}^{\partial MRS(y,Q)}/_{\partial C}-{}^{\partial MRS(y,Q)}/_{\partial\delta}}\\&=-\left.\frac{\partial\delta\left(y,Q\right)}{\partial y}\right|_{\kappa=0}<0.\end{aligned}$$

Donation sensitivity under a given reformed tax schedule is

$$\frac{\partial\delta\left(y,Q\right)}{\partial Q}=\frac{{}^{\partial MRS(y,Q)}/_{\partial Q}}{\mathcal{T}''\left(\delta\left(y,Q\right)\right)+\kappa\mathcal{R}''\left(\delta\left(y,Q\right)\right)+\left(1+\mathcal{T}'\left(\delta\left(y,Q\right)\right)+\kappa\mathcal{R}'\left(\delta\left(y,Q\right)\right)\right){}^{\partial MRS(y,Q)}/_{\partial C}-{}^{\partial MRS(y,Q)}/_{\partial\delta}}>0$$

The sensitivity effect of a tax reform is then

$$\frac{\mathrm{d}}{\mathrm{d}\kappa}\left[\frac{\partial\delta\left(y,Q\right)}{\partial Q}\right]_{\kappa=0}=\mathcal{R}\left(\delta\left(y,Q\right)\right)\frac{\partial\delta\left(y,Q\right)}{\partial Q\partial\mathcal{T}}+\mathcal{R}'\left(\delta\left(y,Q\right)\right)\frac{\partial\delta\left(y,Q\right)}{\partial Q\partial\mathcal{T}'}+\mathcal{R}''\left(\delta\left(y,Q\right)\right)\frac{\partial\delta\left(y,Q\right)}{\partial Q\partial\mathcal{T}''}$$

where

$$\frac{\partial\delta\left(y,Q\right)}{\partial Q\partial\mathcal{T}}\equiv-\left.\frac{\partial^{2}\delta\left(y,Q\right)}{\partial Q\partial y}\right|_{\kappa=0}<0,$$

$$\begin{aligned}\frac{\partial\delta\left(y,Q\right)}{\partial Q\partial\mathcal{T}'}\equiv\left.\frac{\partial\delta\left(y,Q\right)}{\partial\mathcal{T}'}\right|^{C}\frac{\mathrm{d}}{\mathrm{d}\delta}&\left[\frac{{}^{\partial MRS(y,Q)}/_{\partial Q}}{\mathcal{T}''\left(\delta\left(y,Q\right)\right)+\left(1+\mathcal{T}'\left(\delta\left(y,Q\right)\right)\right){}^{\partial MRS(y,Q)}/_{\partial C}-{}^{\partial MRS(y,Q)}/_{\partial\delta}}\right]\\&-\left.\frac{\partial\delta\left(y,Q\right)}{\partial y}\right|_{\kappa=0}\left.\frac{\partial\delta\left(y,Q\right)}{\partial Q}\right|_{\kappa=0}<0,\end{aligned}$$

and

$$\frac{\partial\delta\left(y,Q\right)}{\partial Q\partial\mathcal{T}''}\equiv\left.\frac{\partial\delta\left(y,Q\right)}{\partial\mathcal{T}'}\right|^{C}\left.\frac{\partial\delta\left(y,Q\right)}{\partial Q}\right|_{\kappa=0}<0.$$

Notice, this effect depends on not only how a reform changes the average and marginal tax rate, but also how it changes the second-derivative of the donation tax schedule. Under assumptions 1–4, increases in any of these leads to a decrease in donation sensitivity

At the optimum, it must be the case that for any tax reform $\mathcal{R}$, the first-order welfare effect of a given tax reform is zero:

$$
\begin{aligned}
0 = & \underbrace{\int \left\{ \int_0^\infty \left[1 - \frac{\partial\phi(y_\theta,\xi)/\partial y}{\lambda} - \mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right) \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y} \right] \mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right) \mathrm{d}G\left(\xi\right) \right\} \mu\left(\mathrm{d}\theta\right)}_{\text{redistributive effects}} \\
& \underbrace{- \int \left\{ \int_0^\infty \frac{\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right) \delta\left(y_\theta,\xi\right) \varepsilon\left(y_\theta,\xi\right)}{1+\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)} \mathcal{R}'\left(\delta\left(y,Q\right)\right) \mathrm{d}G\left(\xi\right) \right\} \mu\left(\mathrm{d}\theta\right)}_{\text{efficiency costs}} \\
& \underbrace{+ \left(\frac{\rho}{2\lambda} \int \tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D} \mu\left(\mathrm{d}\theta\right) \right) \int \left\{ \int_0^\infty \left[\frac{\delta\left(y_\theta,\xi\right) \varepsilon\left(y_\theta,\xi\right)}{1+\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)} \mathcal{R}'\left(\delta\left(y,Q\right)\right) + \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y} \mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right) \right] \mathrm{d}G\left(\xi\right) \right\} \mu\left(\mathrm{d}\theta\right)}_{\text{political externality impact via campaign spending effect}} \\
& \underbrace{- \frac{\rho\psi}{\lambda} \int \tau_\theta^{PE} \left\{ \int_0^\infty \left[\mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right) \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial Q \partial \mathcal{T}} + \mathcal{R}'\left(\delta\left(y_\theta,\xi\right)\right) \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial Q \partial \mathcal{T}'} + \mathcal{R}''\left(\delta\left(y_\theta,\xi\right)\right) \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial Q \partial \mathcal{T}''} \right] \mathrm{d}G\left(\xi\right) \right\} \mu\left(\mathrm{d}\theta\right)}_{\text{political externality impact via sensitivity effect}}.
\end{aligned}
$$

After a change of variables, and grouping terms together, we obtain we can equivalently write this condition as

$$
\begin{aligned}
0 = & \int \left(1 - \bar{\omega}\left(\delta\right)\right) \mathcal{R}\left(\delta\right) h\left(\delta\right) \mathrm{d}\delta \\
& - \int \left\{ \left(\frac{\mathcal{T}'\left(\delta\right) - \frac{\rho}{2\lambda} \mathbb{E}\left[\tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]}{1+\mathcal{T}'\left(\delta\right)} \right) \delta\bar{\varepsilon}\left(\delta\right) + \frac{\rho\psi}{\lambda} \mathbb{E}\left[\tau_\theta^{PE} \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial Q \partial \mathcal{T}'} | \delta\left(y_\theta,\xi\right) = \delta \right] \right\} \mathcal{R}'\left(\delta\right) h\left(\delta\right) \mathrm{d}\delta \\
& - \frac{\rho\psi}{\lambda} \int \mathbb{E}\left[\tau_\theta^{PE} \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial Q \partial \mathcal{T}''} | \delta\left(y_\theta,\xi\right) = \delta \right] \mathcal{R}''\left(\delta\right) h\left(\delta\right) \mathrm{d}\delta,
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\omega}\left(\delta\right) \equiv & \frac{\mathbb{E}\left[\partial\phi(y_\theta,\xi)/\partial y | \delta\left(y_\theta,\xi\right) = \delta\right]}{\lambda} + \left(\mathcal{T}'\left(\delta\right) - \frac{\rho}{2\lambda} \mathbb{E}\left[\tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right] \right) \cdot \mathbb{E}\left[\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y} | \delta\left(y_\theta,\xi\right) = \delta \right] \\
& - \frac{\rho\psi}{\lambda} \mathbb{E}\left[\tau_\theta^{PE} \frac{\partial\delta\left(y_\theta,\xi\right)}{\partial Q \partial \mathcal{T}} | \delta\left(y_\theta,\xi\right) = \delta \right].
\end{aligned}
$$

Then, by the fundamental lemma of the calculus of variations, we obtain

$$\int_{\delta}^{\infty} (1-\bar{\omega}(s))\, h(s)\, \mathrm{d}s = \left(\frac{\mathcal{T}'(\delta) - \frac{\rho}{2\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]}{1+\mathcal{T}'(\delta)} \right) \delta\bar{\varepsilon}(\delta)\, h(\delta) + \frac{\rho\psi}{\lambda}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta(y_{\theta},\xi)}{\partial Q\partial\mathcal{T}'} | \delta(y_{\theta},\xi)=\delta\right] h(\delta)$$
$$- \frac{\rho\psi}{\lambda}\frac{\mathrm{d}}{\mathrm{d}\delta}\left\{\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\partial\delta(y_{\theta},\xi)}{\partial Q\partial\mathcal{T}''} | \delta(y_{\theta},\xi)=\delta\right] h(\delta)\right\}$$

Rearranging this expression, we obtain the optimal donation tax formula presented in equation (40).

B Numerical Example

B.1 Deriving a Tractable Optimal Tax Formula

The presence of the convexity effect makes it difficult to apply the necessary condition presented in equation (40) in the task of numerically solving for an optimal tax schedule. To make the optimal tax formula tractable requires an alternative presentation.

To do this, we simplify donation sensitivity using integration by parts:

$$\begin{aligned}\bar{\delta}'_{\theta}(0) &= \int_{0}^{\infty} \delta'_{\theta}(\xi)\, g(\xi)\, \mathrm{d}\xi \\ &= \lim_{\xi\to\infty}\{\delta_{\theta}(\xi)\, g(\xi)\} - \delta_{\theta}(0)\, g(0) - \int_{0}^{\infty}\delta_{\theta}(\xi)\, g'(\xi)\, \mathrm{d}\xi.\end{aligned}$$

By assumption we have $\delta_{\theta}(0) = 0$, and under certain conditions, we can expect that $\lim_{\xi\to\infty}\{\delta_{\theta}(\xi)\, g(\xi)\} = 0$. For example, this will be the case if the preference shock distribution has unbounded support but a finite mean and the donation function $\delta_{\theta}(\cdot)$ is bounded above. Thus, in this case, donation sensitivity for type θ citizens can be written as

$$\bar{\delta}'_{\theta}(0) = -\int_{0}^{\infty}\delta_{\theta}(\xi)\, g'(\xi)\, \mathrm{d}\xi.$$

On the other hand, in certain cases we might expect $\lim_{\xi\to\infty}\{\delta_{\theta}(\xi)\, g(\xi)\} > 0$. For example, if the preference shock distribution has bounded support with positive density at the upper bound of the support ($\bar{\xi}$). In this case, donation sensitivity for type θ citizens can be written as

$$\bar{\delta}'_{\theta}(0) = \delta_{\theta}\left(\bar{\xi}\right) g\left(\bar{\xi}\right) - \int_{0}^{\infty}\delta_{\theta}(\xi)\, g'(\xi)\, \mathrm{d}\xi.$$

Let us consider the first case and assume that $\bar{\delta}'_{\theta}(0) = -\int_{0}^{\infty}\delta_{\theta}(\xi)\, g'(\xi)\, \mathrm{d}\xi$. This alternative presentation of donation sensitivity simplifies the expression for the impact of a tax reform on donation sensitivity considerably:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\kappa}\left[\frac{\partial\delta\left(y,\xi\right)}{\partial Q}\right]_{\kappa=0} &= -\int_0^\infty \left.\frac{\mathrm{d}\delta\left(y,\xi\right)}{\mathrm{d}\kappa}\right|_{\kappa=0} g'\left(\xi\right)\mathrm{d}\xi \\ &= \int_0^\infty \left[\frac{\delta\left(y_\theta,\xi\right)\varepsilon\left(y_\theta,\xi\right)}{1+\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)}\mathcal{R}'\left(\delta\left(y,\xi\right)\right)+\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y}\mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right)\right]\left(\frac{g'\left(\xi\right)}{g\left(\xi\right)}\right)g\left(\xi\right)\mathrm{d}\xi.\end{aligned}$$

Notice, the sensitivity effects of a donation tax reform can now be presented in terms of standard behavioral responses to taxation, though these must be augmented by an additional term capturing the shape of the preference shock distribution $\frac{g'(\xi)}{g(\xi)}$.

Using this new presentation, we can derive an alternative optimality condition:

$$\begin{aligned}0 = &\underbrace{\int\left\{\int_0^\infty\left[1-\frac{\partial\phi(y_\theta,\xi)/\partial y}{\lambda}-\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y}\right]\mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right)\mathrm{d}G\left(\xi\right)\right\}\mu\left(\mathrm{d}\theta\right)}_{\text{redistributive effects}} \\ &\underbrace{-\int\left\{\int_0^\infty\frac{\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)\delta\left(y_\theta,\xi\right)\varepsilon\left(y_\theta,\xi\right)}{1+\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)}\mathcal{R}'\left(\delta\left(y,Q\right)\right)\mathrm{d}G\left(\xi\right)\right\}\mu\left(\mathrm{d}\theta\right)}_{\text{efficiency costs}} \\ &+\underbrace{\left(\frac{\rho}{2\lambda}\int\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\mu\left(\mathrm{d}\theta\right)\right)\int\left\{\int_0^\infty\left[\frac{\delta\left(y_\theta,\xi\right)\varepsilon\left(y_\theta,\xi\right)}{1+\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)}\mathcal{R}'\left(\delta\left(y,Q\right)\right)+\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y}\mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right)\right]\mathrm{d}G\left(\xi\right)\right\}\mu\left(\mathrm{d}\theta\right)}_{\text{political externality impact via campaign spending effect}} \\ &\underbrace{-\frac{\rho\psi}{\lambda}\int\tau_\theta^{PE}\left\{\int_0^\infty\left[\frac{\delta\left(y_\theta,\xi\right)\varepsilon\left(y_\theta,\xi\right)}{1+\mathcal{T}'\left(\delta\left(y_\theta,\xi\right)\right)}\mathcal{R}'\left(\delta\left(y,\xi\right)\right)+\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y}\mathcal{R}\left(\delta\left(y_\theta,\xi\right)\right)\right]\left(\frac{g'\left(\xi\right)}{g\left(\xi\right)}\right)\mathrm{d}G\left(\xi\right)\right\}\mu\left(\mathrm{d}\theta\right)}_{\text{political externality impact via sensitivity effect}}.\end{aligned}$$

Following a change of variables and once again applying the fundamental lemma, we get the alternative optimality condition

$$\int_\delta^\infty\left(1-\tilde{\omega}\left(s\right)\right)h\left(s\right)\mathrm{d}s=\left(\frac{\mathcal{T}'\left(\delta\right)-\frac{\rho}{\lambda}\left(\frac{1}{2}\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]-\psi\mathbb{E}\left[\tau_\theta^{PE}\cdot\frac{\varepsilon(y_\theta,\xi)}{\bar{\varepsilon}(\delta)}\cdot\frac{g'(\xi)}{g(\xi)}|\delta\left(y_\theta,\xi\right)=\delta\right]\right)}{1+\mathcal{T}'\left(\delta\right)}\right)\bar{\varepsilon}\left(\delta\right)\delta h\left(\delta\right), \tag{41}$$

where the social

$$\begin{aligned}\tilde{\omega}\left(s\right)=\frac{\mathbb{E}\left[\partial\phi(y_\theta,\xi)/\partial y|\delta\left(y_\theta,\xi\right)=\delta\right]}{\lambda}+\left(\mathcal{T}'\left(\delta\right)-\frac{\rho\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]}{2\lambda}\right)\cdot\mathbb{E}\left[\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y}|\delta\left(y_\theta,\xi\right)=\delta\right] \\ +\frac{\rho\psi}{\lambda}\mathbb{E}\left[\tau_\theta^{PE}\cdot\frac{\partial\delta\left(y_\theta,\xi\right)}{\partial y}\cdot\frac{g'\left(\xi\right)}{g\left(\xi\right)}|\delta\left(y_\theta,\xi\right)=\delta\right].\end{aligned}$$

B.2 Simulation Procedure

The numerical example presented in section 3.4 is based on the special case where the period 1 utility function for a citizen with quality Q takes the CES form

$$\phi(C,\delta;Q) \equiv \frac{1}{1-\sigma}\left(\left[C^{\frac{e-1}{e}} + Q^{\frac{1}{e}}\delta^{\frac{e-1}{e}}\right]^{\frac{e}{e-1}}\right)^{1-\sigma}.$$

In this special case, some simplifications can be made to the optimal donation tax formula. For example, it can be shown that

$$\begin{aligned}\varepsilon(y_\theta,\xi)\,\delta h(\delta|\theta) &= \frac{\varepsilon(y_\theta,\xi)}{\frac{\xi_\theta^{-1}(\delta)}{\delta}\frac{\partial\delta\left(y_\theta,\xi_\theta^{-1}(\delta)\right)}{\partial Q}}\xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right)\\ &= e\xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right),\end{aligned}$$

where $\xi_\theta^{-1}(x) \equiv \{\xi > 0 : \delta(y_\theta,\xi) = x\}$. Using this convenient relationship, equation (41) can be simplified to

$$\frac{\int_\delta^\infty (1-\tilde{\omega}(s))\, h(s)\,\mathrm{d}s}{\int \xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)} = \left(\frac{\mathcal{T}'(\delta) - \frac{\rho}{\lambda}\left\{\frac{1}{2}\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right] + \psi\cdot\frac{\int\left[\tau_\theta^{PE}\left(1+\alpha\left(\xi_\theta^{-1}(\delta)\right)\right)\right]g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)}{\int \xi_\theta^{-1}(\delta)g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)}\right\}}{1+\mathcal{T}'(\delta)}\right)\cdot e.$$

Further note that in the example, I have assumed that there are a finite number of citizen types. Consequently, there exists a maximum possible donation amount δ^{max}, since the most a citizen can feasibly donate is 100% of their income. I have also assumed that the preference shock distribution is normal with a standard deviation b: $G(\xi) \equiv \Phi\left(\frac{\xi}{b}\right)$.

Optimal Top Tax Rate

Given these assumptions, let's find the optimal top tax rate at $\mathcal{T}'(\delta_{max})$. Note that

$$\begin{aligned}\lim_{\delta\to\delta_{max}}\left\{\frac{\int_\delta^{\delta_{max}}(1-\tilde{\omega}(s))\,h(s)\,\mathrm{d}s}{1-H(\delta)}\right\} &= \frac{\lim_{\delta\to\delta_{max}}(1-\tilde{\omega}(\delta))\cdot h(\delta)}{\lim_{\delta\to\delta_{max}} h(\delta)}\\ &= 1-\tilde{\omega}(\delta_{max}).\end{aligned}$$

As well,

$$\begin{aligned}
\lim_{\delta \to \delta_{max}} \left\{ \frac{1 - H(\delta)}{\int \xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)} \right\} &= \lim_{\delta \to \delta_{max}} \left\{ \frac{1 - H(\delta)}{\int \xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)} \right\} \\
&= \lim_{\delta \to \delta_{max}} \left\{ \frac{1 - G\left(\xi_{\theta_{max}}^{-1}(\delta)\right)}{\xi_{\theta_{max}}^{-1}(\delta)\, g\left(\xi_{\theta_{max}}^{-1}(\delta)\right)} \right\} \\
&= \lim_{\xi \to \infty} \left\{ \frac{1 - \Phi\left(\frac{\xi}{b}\right)}{\frac{\xi}{b}\phi\left(\frac{\xi}{b}\right)} \right\} \\
&= 0
\end{aligned}$$

where θ_{max} is the citizen type with the highest income.[26]

Given the results thus far, the optimal top tax rate formula reduces to

$$\mathcal{T}'(\delta_{max}) = \frac{\rho}{\lambda} \left\{ \frac{1}{2}\mathbb{E}\left[\tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right] + \psi \cdot \frac{\int \left[\tau_\theta^{PE}\left(1 + \alpha\left(\xi_\theta^{-1}(\delta)\right)\right)\right] g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)}{\int \xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)} \right\}$$

Note, under the assumption of normally distributed preference shocks we have $\frac{g'(\xi)}{g(\xi)} = -\frac{\xi}{b^2}$. Consequently,

$$\begin{aligned}
\lim_{\delta \to \delta_{max}} \left\{ \frac{\int \left[\tau_\theta^{PE}\left(1 + \alpha\left(\xi_\theta^{-1}(\delta)\right)\right)\right] g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)}{\int \xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)} \right\} &= -\lim_{\delta \to \delta_{max}} \left\{ \frac{\int \tau_\theta^{PE} \xi_\theta^{-1}(\delta)\, g'\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)}{\int \xi_\theta^{-1}(\delta)\, g\left(\xi_\theta^{-1}(\delta)\right) \mu(\mathrm{d}\theta)} \right\} \\
&= -\lim_{\delta \to \delta_{max}} \left\{ \frac{\tau_{\theta_{max}}^{PE} \xi_{\theta_{max}}^{-1}(\delta)\, g'\left(\xi_{\theta_{max}}^{-1}(\delta)\right)}{\xi_{\theta_{max}}^{-1}(\delta)\, g\left(\xi_{\theta_{max}}^{-1}(\delta)\right)} \right\} \\
&= \frac{\tau_{\theta_{max}}^{PE}}{b^2} \lim_{\xi \to \infty} \{\xi\} \\
&= \infty
\end{aligned}$$

Thus, the optimal top tax rate on donations diverges to infinity:

$$\mathcal{T}'(\delta_{max}) = \infty.$$

[26]Note, the last line follows from the properties of the normal distribution, which is not heavy-tailed.

Solving for Other Rates

For other tax rates, we need to derive them via simulation. Rearranging the optimal tax formula, we obtain the following expression:

$$\mathcal{T}'(\delta) = \frac{\frac{1}{e}\frac{\int_{\delta}^{\delta^{\max}}(1-\tilde{\omega}(s))h(s)\mathrm{d}s}{\int \xi_\theta^{-1}(\delta)g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)} + \frac{\rho}{\lambda}\left\{\frac{1}{2}\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right] + \psi\cdot\frac{\int\left[\tau_\theta^{PE}\left(1+\alpha\left(\xi_\theta^{-1}(\delta)\right)\right)\right]g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)}{\int \xi_\theta^{-1}(\delta)g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)}\right\}}{1-\frac{1}{e}\frac{\int_{\delta}^{\delta^{\max}}(1-\tilde{\omega}(s))h(s)\mathrm{d}s}{\int \xi_\theta^{-1}(\delta)g\left(\xi_\theta^{-1}(\delta)\right)\mu(\mathrm{d}\theta)}} \tag{42}$$

I numerically solve for a discrete approximation to the optimal tax schedule which specifies the marginal tax rate at a finite number of donation amounts. I begin with some initial schedule $\mathcal{T}_0$ and compute an approximation to the righthand side of equation (42) under this schedule. This yields an updated marginal tax rate at each donation amount I consider in my approximation. I iterate on this procedure until convergence is achieved.

Specifically, I implement the following procedure:

1. Pick some fixed value for the multiplier on the social planner's budget constraint λ.

2. Pick some $N_\delta \in \mathbb{N}$. Initialize tax rates to $\mathcal{T}_0'(\delta) = \mathcal{T}_0'$ and $\mathcal{T}_0(\delta) = (1+\mathcal{T}_0')\,\delta$ for all $\delta \in \{\delta_0, \delta_1, \delta_2, \ldots, \delta_{N_\delta-1}, \delta_{N_\delta}\}$, where $\delta_0 = 0$ and $\delta_{N_\delta} = \max y_\theta$ is the maximum possible donation amount.

3. Set $k = 0$.

4. For convenience, let

$$\xi_k^{-1}(\delta, y_\theta) \equiv \left(\frac{\delta}{y_\theta - \delta - \mathcal{T}_k(\delta)}\right)\left(1+\mathcal{T}_k'(\delta)\right)^e$$

$$\hat{G}_k(\delta, y_\theta) \equiv G\left(\xi_k^{-1}(\delta, y_\theta)\right),$$

$$\hat{g}_k(\delta, y_\theta) \equiv g\left(\xi_k^{-1}(\delta, y_\theta)\right),$$

and

$$\hat{\alpha}_k(\delta, y_\theta) \equiv -\frac{\xi_k^{-1}(\delta, y_\theta)\,g'\left(\xi_k^{-1}(\delta, y_\theta)\right)}{g\left(\xi_k^{-1}(\delta, y_\theta)\right)}.$$

With a normal distribution, $\hat{\alpha}_k(\delta, y_\theta) = \left(\frac{\xi_k^{-1}(\delta, y_\theta)}{b}\right)^2$.

5. Compute total donations via trapezoidal approximation:

$$D_k = \frac{1}{2}\sum_{n=1}^{N_\delta}(\delta_n - \delta_{n-1})\left[\sum_\theta \pi_\theta\left\{\left(1-\hat{G}_k(\delta_{n-1}, y_\theta)\right) + \left(1-\hat{G}_k(\delta_n, y_\theta)\right)\right\}\right]$$

6. Compute total tax revenue via trapezoidal approximation:

$$Rev_k = \sum_{n=1}^{N_\delta} (\delta_n - \delta_{n-1}) \left(\mathcal{T}_k' (\delta_n) \left[1 - \sum_\theta \pi_\theta \hat{G}_k (\delta_{n-1}, y_\theta) \right] + \mathcal{T}_k' (\delta_{n-1}) \left[1 - \sum_\theta \pi_\theta \hat{G}_k (\delta_n, y_\theta) \right] \right)$$

7. Compute donation sensitivity for each citizen type via trapezoidal approximation:[27]

$$\bar{\delta}_{\theta,k}' (0) = \frac{1}{2} \sum_{n=1}^{N_\delta} (\delta_n - \delta_{n-1}) \left(\hat{g}_k (\delta_{n-1}, y_\theta) + \hat{g}_k (\delta_n, y_\theta) \right)$$

8. Compute the marginal effect of campaign spending:

$$\psi_k = \sum_\theta \pi_\theta \eta_\theta' (D_k)$$

9. Compute welfare weights for each citizen type:

$$\gamma_{\theta,k} = 2\eta_\theta (D_k) \, g(0) + \psi_k \bar{\delta}_{\theta,k}' (0) \, .$$

10. Compute equilibrium policy by solving for $t_{y,k}^\star$:

$$t_{y,k}^\star = \arg \max_{t_y \in [0,1]} \sum_\theta \pi_\theta \left\{ \frac{(y_\theta + t_y (\bar{y} - y_\theta))^{1-\sigma}}{1-\sigma} \right\} .$$

11. Compute political externality wedges for each citizen type:

$$\tau_{\theta,k}^{PE} = -\frac{1}{\sigma} \left(\frac{\sum_{\theta'} \pi_{\theta'} \left\{ \frac{\bar{y} - y_{\theta'}}{\left(y_{\theta'} + t_{y,k}^\star (\bar{y} - y_{\theta'})\right)^\sigma} \right\}}{\sum_{\theta'} \pi_{\theta'} \left\{ \frac{\gamma_{\theta',k} (\bar{y} - y_{\theta'})^2}{\left(y_{\theta'} + t_{y,k}^\star (\bar{y} - y_{\theta'})\right)^{\sigma+1}} \right\}} \right) \frac{\bar{y} - y_\theta}{\left(y_\theta + t_{y,k}^\star (\bar{y} - y_\theta) \right)^\sigma}$$

12. Compute $\mathbb{E}\left[\tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]$:

$$\mathbb{E}\left[\tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]_k = \sum_\theta \pi_\theta \tau_{\theta,k}^{PE} \frac{\mathrm{d}\gamma_{\theta,k}}{\mathrm{d}D_k}$$

[27]

$$\begin{aligned} \bar{\delta}_\theta' (0) &= \int_0^\infty \delta_\theta' (\xi) \, g(\xi) \, \mathrm{d}\xi \\ &= \int_0^{\delta^{max}} \delta_\theta' \left(\xi_\theta^{-1} (\delta, y_\theta) \right) h(\delta|\theta) \, \mathrm{d}\delta \\ &= \int_0^{\delta^{max}} g \left(\xi_\theta^{-1} (\delta, y_\theta) \right) \mathrm{d}\delta \end{aligned}$$

where

$$\frac{\mathrm{d}\gamma_{\theta,k}}{\mathrm{d}D_k} = 2g\left(0\right)\eta_\theta'\left(D_k\right) + \bar{\delta}_{\theta,k}'\left(0\right) \cdot \sum_{\theta'} \pi_{\theta'}\eta_{\theta'}''\left(D_k\right).$$

13. Compute $\Omega\left(\delta\right) \equiv \int_\delta^{\delta^{\max}} \left(1 - \tilde{\omega}\left(s\right)\right) h\left(s\right) \mathrm{d}s$:

$$\begin{aligned}\Omega_k\left(\delta_m\right) = \Omega_k^1\left(\delta_m\right) + \frac{1}{\lambda}\sum_{n=m+1}^{N_\delta}\Omega_k^2\left(\delta_n\right) + \frac{1}{2}\sum_{n=m+1}^{N_\delta}\left(\delta_n - \delta_{n-1}\right)\left(\Omega_k^{3A}\left(\delta_n\right) + \Omega_k^{3A}\left(\delta_{n-1}\right)\right) \\ + \frac{1}{\lambda}\frac{1}{2}\sum_{n=m+1}^{N_\delta}\left(\delta_n - \delta_{n-1}\right)\left(\Omega_k^{3B}\left(\delta_n\right) + \Omega_k^{3B}\left(\delta_{n-1}\right)\right) \\ + \frac{1}{\lambda}\frac{1}{2}\sum_{n=m+1}^{N_\delta}\left(\delta_n - \delta_{n-1}\right)\left(\Omega_k^{4}\left(\delta_n\right) + \Omega_k^{4}\left(\delta_{n-1}\right)\right)\end{aligned}$$

where

$$\Omega_k^1\left(\delta_m\right) = 1 - \sum_\theta \pi_\theta \hat{G}_k\left(\delta_m, y_\theta\right)$$

$$\Omega_k^2\left(\delta_n\right) = -\sum_\theta \pi_\theta \frac{\left(y_\theta - \delta_n - \mathcal{T}_k\left(\delta_n\right) + \left(1 + \mathcal{T}_k'\left(\delta_n\right)\right)\delta_n\right)^{\frac{1-\sigma e}{e-1}}}{\left(y_\theta - \delta_n - \mathcal{T}_k\left(\delta_n\right)\right)^{\frac{1-\sigma}{e-1}}}\left(\hat{G}_k\left(\delta_n, y_\theta\right) - \hat{G}_k\left(\delta_{n-1}, y_\theta\right)\right)$$

$$\Omega_k^{3A}\left(\delta_m\right) = -\mathcal{T}_k'\left(\delta\right) \cdot \sum_\theta \pi_\theta \left[\frac{\xi_k^{-1}\left(\delta, y_\theta\right)}{y_\theta - \delta - \mathcal{T}_k\left(\delta\right)}\right]\hat{g}_k\left(\delta, y_\theta\right)$$

$$\Omega_k^{3B}\left(\delta_m\right) = \frac{\rho}{2}\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]_k \sum_\theta \pi_\theta \left[\frac{\xi_k^{-1}\left(\delta, y_\theta\right)}{y_\theta - \delta - \mathcal{T}_k\left(\delta\right)}\right]\hat{g}_k\left(\delta, y_\theta\right)$$

$$\Omega_k^4\left(\delta\right) = \frac{\rho\psi_k}{2} \cdot \sum_\theta \pi_\theta \left[\tau_{\theta,k}^{PE}\frac{\hat{\alpha}_k\left(\delta, y_\theta\right)}{y_\theta - \delta - \mathcal{T}_k\left(\delta\right)}\right]\hat{g}_k\left(\delta, y_\theta\right).$$

14. Update marginal tax rates for all $\delta \in \{\delta_0, \delta_1, \ldots, \delta_{N_\delta - 1}\}$,

$$\mathcal{T}_{k+1}'\left(\delta\right) = \min\left\{\frac{\frac{\Omega_k(\delta)}{\overline{\xi g}_k(\delta)} + e\frac{\rho}{2\lambda}\left\{\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]_k + \psi_k\frac{\overline{\tau\alpha g}_k(\delta)}{\overline{\xi g}_k(\delta)}\right\}}{e - \frac{\Omega_k(\delta)}{\overline{\xi g}_k(\delta)}}, \overline{\mathcal{T}'}\right\}$$

where

$$\overline{\xi g}_k\left(\delta\right) = \sum_\theta \pi_\theta \xi_k^{-1}\left(\delta, y_\theta\right)\hat{g}_k\left(\delta, y_\theta\right),$$

$$\overline{\tau\alpha g}_k\left(\delta\right) = \sum_\theta \pi_\theta \tau_\theta^{PE}\hat{\alpha}_k\left(\delta, y_\theta\right)\hat{g}_k\left(\delta, y_\theta\right),$$

and $\overline{\mathcal{T}'} > 0$ is some positive real number which is the highest possible marginal tax rate for the purpose of my simulations. Adopting an upper limit on marginal rates is necessary to address numerical issues which arise as a result of the divergence of the optimal top tax rate. For the marginal tax rate on the highest possible income, I simply impose that $\mathcal{T}'_{k+1}(\delta_{N_\delta}) = \overline{\mathcal{T}'}$, as an approximation to the infinite optimal top tax rate.

15. Update tax liabilities for all $\delta \in \{0, \delta_1, \ldots, \delta_{N_\delta}\}$. I always impose the restriction that

$$\mathcal{T}_{k+1}(0) = 0.$$

For other donation amounts, we have

$$\mathcal{T}_{k+1}(\delta_m) = \frac{1}{2}\sum_{n=1}^{m}(\delta_n - \delta_{n-1})\left(\mathcal{T}'_{k+1}(\delta_n) + \mathcal{T}'_{k+1}(\delta_{n-1})\right)$$

16. Check for convergence. If converged, stop. If not converged, set $k = k + 1$ and return to step 3.